Trajectory and Vibration Control of a Single-Link Flexible-Joint Manipulator Using a Distributed Higher-Order Differential Feedback Controller

The trajectory tracking in the flexible-joint manipulator (FJM) system becomes complicated since the flexibility of the joint of the FJM superimposes vibrations and nonminimum phase characteristics. In this paper, a distributed higher-order differential feedback controller (DHODFC) using the link and joint position measurement was developed to reduce joint vibration in step input response and improve tracking behavior in reference trajectory tracking control. In contrast to the classical higher-order differential (HOD), the dynamics of the joint and link are considered separately in DHODFC. In order to validate the performance of the DHODFC, step input, trajectory tracking, and disturbance rejection experiments are conducted. In order to illustrate the differences between classical HOD and DHODFC, the performance of these controllers is compared based on tracking errors and energy of control signal in the tracking experiments and fundamental dynamic characteristics in the step response experiments. DHODFC produces better tracking errors with almost same control effort in the reference tracking experiments and a faster settling time, less or no overshoot, and higher robustness in the step input experiments. Dynamic behavior of DHODFC is examined in continuous and discontinuous inputs. The experimental results showed that the DHODFC is successful in the elimination of the nonminimum phase dynamics, reducing overshoots in the tracking of such discontinuous input trajectories as step and square waveforms and the rapid damping of joint vibrations. [DOI: 10.1115/1.4035873]

Keywords: flexible-joint manipulator, higher-order differential feedback controller, distributed higher-order differential feedback controller, trajectory tracking, differential flatness

1 Introduction

Flexible robotic manipulators were studied first by researchers in space exploration robots [1]. The higher levels of maneuverability combined with the relatively lower levels of power consumption of flexible-joint manipulators (FJMs) when compared to rigid-body robotics have continued to attract the use of these flexible systems in specialized applications. The trend in the robotics industry is to design lightweight robots with a high payload, especially in the fields, such as surgical, space, and service robotics [2]. This new concept leads to consider flexibility in the robot joints.

In the literature, feedback control methods related to joint flexibility have been proposed numerous. Some of them require accurate model and computational complexity. Hence, they produce very powerful theoretical results about control of FJM. Implementation of these model-based control methods may be very difficult [2]. Because of this reason, model-free controller approaches attract attention nowadays. The existence of a very significant amount of related literature dealing with problems of modeling, nonlinearities, frictional effects, and model uncertainties [3–10] could probably be attributed to the need to improve the performance of these systems for the large range of practical applications that have been developed since the 1990s. A detailed literature review about dynamic analyses of flexible robotic manipulators can be found in Ref. [1]. Yavuz et al. [7] analyzed how residual vibrations of flexible systems can be eliminated and proposed hybrid input shaping technique. Talole et al. [8] presented an extended state observer based feedback linearization method for the trajectory tracking control of a flexible-joint robotic manipulator. They designed a model-based state estimator and validated experimentally.

Over the last two decades, the control of FJMs has also benefited from strategies that seek to reduce the impact of modeling difficulties on controller performance, instead making use such nonmodel intensive strategies as genetic algorithms, particle swarm optimization methods, fuzzy logic, neural networks [11–17], and more recently, the so-called iPI and iPID strategies [18–20]. These pseudomodel based control strategies have enabled a compromise between the cost of real-time controller implementation and the need for highly accurate models in model-based control. A pseudomodel technique recently applied to the control of FJMs, the higher-order differential feedback controller (HODFC), was reported to have achieved very good trajectory control [21]. The approach that forms the body of the current contribution is a further exploitation of the HODFC structure for achieving further performance improvements in the control of the manipulator. Typically, the HODFC strategy requires only knowledge of the model-order for controller design. The HODFC strategy does not need the actual model parameters for controller implementation.

This paper is organized as follows: a recollection of the system model is presented in Sec. 2. The derivation of the flat output
based model for the flexible-joint system, together with use of the flat output to explain the impact of joint flexibility and that of the associated nonminimum phase dynamics, is also included in Sec. 2. A recollection of the design of the higher-order differential feedback control strategy and the design of the distributed HODFC structure for the manipulator is presented in Sec. 3. In Sec. 4, laboratory experiments validating the performance of the distributed HODFC controller are presented and discussed. Also, the experimental results of the distributed HODFC are compared with results obtained from controlling the system with the classical HODFC structure. The overall conclusions are given in Sec. 5.

2 System Modeling and Experimental Setup

Figure 1 shows a picture of the FJM system used in this study. The experimental setup consists of a real-time control platform (dspace ds1103), a computer, a geared brushless DC motor (Faulhaber 2444S), a motor driver (Faulhaber MCBL5004), a rotary encoder (2000 P/R), two springs (58.86 N/m), and a rigid link (0.4 m).

A model of the system based on derivation in Ref. [22] is recollected in this section. An alternative presentation of the system model in terms of a flat output and the use of the flat output to clarify the existence and problems of nonminimum phase dynamics in the manipulator are also included.

2.1 Mathematical Modeling of the Flexible-Joint Manipulator. Three-dimensional solid model of the physical system is shown in Fig. 2. The detailed modeling of the FJM has been presented severally in literature. Specifically, it has been shown in Ref. [22] that the Lagrangian (L) of the single-link FJM system is given below

\[ L = T - V \]

\[ L = J_1 \dot{\theta}^2 + \frac{1}{2} I (\dot{\theta} + \dot{z})^2 - \frac{1}{2} K_s \dot{z}^2 \]  

(1)

where \( \theta \) and \( z \) are the generalized system coordinates, \( J_1 \) is the system’s moment of inertia, \( I \) is the moment of inertia of the link, and \( K_s \) is the resulting spring constant derived from the two springs forming the flexible joint. Moreover, the potential energy of the system was obtained to yield \( V = K_s \dot{z}^2 / 2 \), and the kinetic energy is \( T = 1/2 (J_1 \dot{\theta}^2 + I (\dot{\theta} + \dot{z})^2) \). The Euler–Lagrangian derivation of the system model [22] then produces two second-order linear systems given by

\[ J_1 \ddot{\theta} + I (\ddot{\theta} + \ddot{z}) = \tau - B \dot{\theta} \]

\[ I (\ddot{\theta} + \ddot{z}) + K_s \dot{z} = 0 \]  

(2)

where \( B \) is the constant of viscous damping acting in the drive system. With the much faster dynamics of the current in the drive motor neglected, the expression for the torque \( \tau \) [11] delivered by the drive system is given by the following equation:

\[ \tau = -\frac{K_s K_m K_g}{R_m} \dot{\theta} + \frac{K_s K_g}{R_m} v(t) \]  

(3)

where \( v(t) \) is the drive voltage applied to the motor, \( R_m \) is the motor armature resistance, \( K_m \) is the back emf constant of the drive motor, \( K_s \) is the motor torque’s constant, and \( K_g \) is the gear ratio.

2.2 An Equivalent Flat Representation of the Dynamics of the Manipulator. It was shown in Ref. [21] that there is a variable \( y \), defined as

\[ y(t) = \dot{\theta}(t) + \alpha(t) \]  

(4)

such that, if one differentiates Eq. (4) twice and substitutes the result in the second component of Eq. (2), then one obtains that

\[ \alpha = -\frac{J_1}{K_s} \dot{y}, \quad \ddot{\alpha} = -\frac{J_1}{K_s} \dot{y}^3, \quad \dddot{\alpha} = -\frac{J_1}{K_s} \dot{y}^4 \]  

(5)

Combining Eqs. (4) and (5) leads to the following equation:

\[ \theta = y + \frac{I}{K_s} \dot{y}, \quad \dot{\theta} = \dot{y} + \frac{I}{K_s} \dot{y}^3, \quad \ddot{\theta} = \ddot{y} + \frac{I}{K_s} \dot{y}^4 \]  

(6)

To facilitate some of the analysis to be presented later in the paper, one could rewrite from Eqs. (5) and (6) that

\[ \alpha(s) = \frac{s^2}{K_s} Y(s) \]

\[ \theta(s) = \left(\frac{K_s + s^2 I}{K_s}\right) Y(s) \]  

(7)

Now, using the relevant components of Eqs. (5) and (6) together with Eq. (3), a fourth-order model for the FJM, with a bounded disturbance \( d(t) \), could be written as follows:

\[ y^{(4)} = f(y) + b_0 v(t) + d(t) \]

\[ f(y) = -\frac{BR_m + K_m K_s y^3}{J_1 R_m} - \frac{J_1 K_m R_m + I K_s R_m}{J_1 R_m} \dot{y} - \frac{K_s B R_m + K_s K_m K_g^2}{J_1 R_m} \ddot{y} \]

\[ b_0 = \frac{K_s K_g}{J_1 R_m} \]  

(8)

The variable \( y \), which could be used with its derivatives to achieve an alternative representation of a given dynamic system, is said to be the flat output of the system [21]. In Ref. [21], the form was used for the implementation of the HODFC for the system. Here, Eqs. (5)–(8) are used first to motivate the need for the distributed structure of the HODFC controller, which is the subject of this article.

2.3 The Nonminimum Phase Dynamics of the Flexible-Joint Manipulator. Equations (5)–(8) provide a quantitative way of assessing the impact of the nonminimum phase dynamics of the flexible joint in degrading controller performance. Thus, rewriting Eq. (8) in the transfer function form to yield the following equation:

\[ Y(s) = \frac{b_0 V(s)}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \]

\[ a_3 = \frac{B R_m + K_m K_s y^3}{J_1 R_m}; \quad a_2 = J_1 K_m R_m + I K_s R_m; \]

\[ a_1 = \frac{K_s B R_m + K_s K_m K_g^2}{J_1 R_m} \]  

(9)

![Fig. 1 FJM experimental setup](image-url)
Then, a simple substitution of Eq. (9) into the two parts of Eq. (7), respectively, yields

\[ \theta(s) = \frac{b_0(K_s + I_s^2)}{K_s(s^2 + a_3s^2 + a_2s + a_1)} V(s) \] (10)

\[ x(s) = -\frac{s^2h_0}{K_s(s^2 + a_3s^2 + a_2s + a_1)} V(s) \] (11)

For the sake of the HODF controller design, Eq. (10) could be rewritten as

\[ K_s \left( \frac{d^2\theta}{dt^2} + a_3 \frac{d\theta}{dt} + a_2 \frac{d\theta}{dt} + a_1 \theta \right) = b_0K_s\dot{v}(t) + b_0h_0 \frac{d^2\dot{v}(t)}{dt^2} \]

\[ \dot{\theta} = f(\theta) + b_0\dot{v}(t) + b_0h_0 \frac{d^2\dot{v}(t)}{dt^2} \]

\[ f(\theta) = -\{a_3\dot{\theta} + a_2\dot{\theta} + a_1\dot{\theta} \} \] (12)

In a similar fashion, it is straightforward to show that

\[ \dot{z} = f(z) - \frac{b_0h_0 \frac{d^2\dot{v}(t)}{dt^2}}{K_s} \]

\[ f(z) = -\{a_3z + a_2\dot{z} + a_1\dot{z} \} \] (13)

Equation (13) quantifies the nonminimum phase part of the dynamics of the manipulator.

2.4 Nonminimum Phase Dynamics and System Measurements. Now, define a new system variable as follows:

\[ H(s) = \frac{b_0}{s^3 + a_3s^2 + a_2s + a_1} \] (14)

if \( a_3a_2 - a_1 > 0 \) (determined using the Routh–Hurwitz criterion), the impulse response of \( H(s) \) given below

\[ h(t) = L^{-1}\left\{ \frac{b_0}{s^3 + a_3s^2 + a_2s + a_1} \right\} \] (15)

is stable. And from Eq. (9), one could use the convolution integral to write that

\[ y(t) = \int_0^t \left( \int_0^\infty h(\tau)v(t - \tau)d\tau \right) dt \] (16)

and Eqs. (10) and (11) combined with Eq. (16) yield

\[ \theta(t) = y(t) + \int \frac{I}{K_s} \int_0^\infty h(\tau)v(t - \tau)d\tau dt \] (17)

if the linear operators (differentiation and integration) in Eq. (17) are interchanged, then, one obtains

\[ \theta(t) = \left\{ \begin{array}{ll} y(t) + \int \frac{I}{K_s} \int_0^\infty h(\tau) \frac{d}{dt} v(t - \tau) d\tau, & 0 < t < \infty \\ y(t), & t \rightarrow \infty \end{array} \right. \] (18)

Similarly,

\[ \alpha(t) = \left\{ \begin{array}{ll} -\frac{1}{K_s} \int_0^\infty h(\tau) \frac{d}{dt} v(t - \tau) d\tau, & 0 < t < \infty \\ 0, & t \rightarrow \infty \end{array} \right. \] (19)

Note that the -180 deg shift in the phase of the dynamics of joint of the manipulator (\( \alpha(t) \)), as shown in Eq. (19), accounts for the joint oscillations commonly observed on the practical flexible-joint system. It is also evident from Eq. (19) that the nature of these oscillations is determined by the derivative content of the input signal \( v(t) \); this fact also bears out well in experimental systems. The negative phase shift also accounts for the out-of-phase dynamics of the joint, which significantly adds to the initial tracking error. It is further evident from Eq. (19) that the direct measurement of \( \alpha(t) \) would provide sufficient derivative content in the measurement, to enable a controller to damp the joint oscillations.

The following general remarks could further be made on the basis of Eqs. (16)–(19):

(i) Equations (18) and (19) confirm the validity of Eq. (4).

(ii) The dynamics of \( \theta(t) \) and \( \alpha(t) \) contain the differential of the input trajectory signal \( v(t) \). For trajectories with finite number of discontinuities, both \( \theta(t) \) and \( \alpha(t) \) would experience instantaneous jumps in values.

(iii) The corollary to (ii) is that because \( \theta(t) \) and \( \alpha(t) \) are reached in the derivative information contained in the input signal \( v(t) \), their direct measurement for use in control would provide the additional information for the controller to keep tracking such rapidly varying signals as square-wave trajectories.

(iv) A consequence of the joint flexibility is that, when combined, the derivative content in \( \alpha(t) \) filters the derivative information available in \( \theta(t) \). Consequently, the traditional use of the output \( y(t) = \theta(t) + \alpha(t) \) does not provide the derivative information required to effectively track discontinuous or rapidly varying trajectories.

2.5 Nonminimum Phase and Error Signal. If the system is required to track a reference trajectory \( \theta_r(t) \), then the error generated from measuring \( y(t) \) for feedback is given by

\[ e(t) = \theta_r(t) - y(t) = \{ \theta_r(t) - \theta(t) \} + \alpha(t) \to \infty \]

\[ e(t) \to \theta_r(t) - \theta(t); t \to \infty \] (20)
Again, the nonminimum phase dynamics lead to initial error computation that is not very indicative of the error between the output trajectory and the reference trajectory. In fact, the initial error generated from the measurements contains a positive feedback term due to $z(t)$. This definitely constitutes a major control system design challenge when $y(t)$ is used as the feedback signal.

### 2.6 Summary of the Consequences of Joint Flexibility

It could be summarized that joint flexibility in the manipulator filters out critical trajectory information from the measurements, such that the traditional design of controllers based on the overall system measurement $y(t)$ does not provide the controller with sufficient information for tracking trajectories with significant derivative content. Therefore, there is motivation to use the direct measurement of $\theta(t)$ and $\phi(t)$, which contain significant information about the trajectory being tracked, for controller design. This employment of differential measurement for the control of the FJM leads to the concept of distributed control, as utilized in this paper. In the Sec. 3, a recollection of the classical HODFC used in Ref. [21] is presented. The mathematical formulation of the structure of the distributed HODFC controller then follows.

### 3 The Higher-Order Differential Feedback Controller

A substantial literature exists about HODF controller [23,24]. Figure 3, used in Ref. [21], summarizes the structure of the HODFC controller. The classical HODFC consists of two higher-order differential (HOD) structures, feedback gains, and a filter with a very high forward gain. The HOD structures generate appropriate derivatives which are used for loop closure.

To design an HOD system for a system of $n$th order, the HOD is an $m$th order system with two model-free parameters $m$ and $c_0$. Specifically, the $m$th ($\geq n+1$) order of the HOD is first decided, then the parameter $c_0$ is selected between 2 and 50 [23,24]. The following parameters are then evaluated:

$$K_{\text{hod}} = \frac{m^m}{(m - 1)!^{m-1}} \quad (21)$$

$$c_i = c_0^{-i}K_{\text{hod}}c_{m-1}^{-1}; \quad i = 1, 2, ..., m$$

where $C_i$ denotes the combination expression. The $c_i$ ($1 \leq i \leq m$) parameters are required for the implementation of the HOD system. If an HOD system designed to process the measurement $y(t)$ for a system of order $n$ is considered, then $m$ integrators realizing the HOD are given by [23,24] the following equation:

$$z_i = z_{i+1} + c_i(\psi - z_i), \quad 1 \leq i \leq m - 1$$

$$z_m = c_m(\psi - z_1)$$

$$\psi = y + w(t)$$

where $z_1, z_2, ..., z_n$ are the outputs of the $m$-integrators forming the HOD system, and $\psi(t)$ is the measurement of $y(t)$, corrupted by the measurement noise $w(t)$. Provided the conditions in Eq. (21) are satisfied, the HOD yields accurate estimates of the variable $y(t)$ as given by Qi et al. [23] by the following equation:

$$\ddot{z} = z_1$$

$$\dot{z}_i = z_{i+1} + c_i(\psi - z_i), \quad i = 1, 2, ..., n \quad (23)$$

and

$$\lim_{t \to \infty} \dot{z}_i = y_i$$

As the HOD has an order $m > n$, the convergence of the $m$th-order HOD output guarantees the accuracy of the estimates of the $n$th-order system being controlled. The stability and convergence properties of the HOD and that of the estimated parameters have been severally discussed in detail in the existing literature [23,24].

#### 3.1 Classical Higher-Order Differential Feedback Controller

Now, suppose that the output of the affine system $(y(t)) = f(y) + g(y)a(t)$ is required to track a trajectory $y_r(t)$. If all the derivatives of both $y_r(t)$ and $y(t)$ are available, then it is straightforward to generate the error variable and its derivatives such that

$$e = y_r - y$$

$$\dot{e} = \dot{y}_r - \dot{y}$$

$$e^{(n)} = y^{(n)} - y^{(n)}_r = y^{(n)} - y^{(n)} + y^{(n)} - (f(y) + g(y)a + d(t))$$

(24)

Hence, for the first part of the HODFC controller design, set in Eq. (24)

$$y^{(n)} - y^{(n)}_r = k_0e + k_1e_2 + ... + k_{n-1}e_n = Ke$$

(25)

where the elements of vector $K$ have been chosen such that

$$0 = (k_0 + k_1s + k_2s^2 + ... + k_{n-1}s^{n-1} + s^n)$$

(26)

is Hurwitz. The pole-placement structure in Eq. (26) is the first part of the HODFC. In practical HODFC implementation, the error states used are obtained as estimates, $e$ from the HOD. However, the practice is to leave the notation of Eq. (24) as it is, for ease of use. Now, combine Eqs. (24) and (26) to obtain

$$e = y_r - y$$

$$\dot{e} = \dot{y}_r - \dot{y}$$

$$e^{(n)} = -k_0e - k_1e_2 - ... - k_{n-1}e_n + y^{(n)} - (f(y) + g(y)a + d(t))$$

(27)
For the complete controller structure, set
\[ y^{(e)} - (f(y) + g(y)u(t) + d(t)) = \dot{u} \] (28)

The HODFC controlled system yields
\[\begin{align*}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= e_3 \\
\vdots \\
\dot{e}_n &= -Ke + \ddot{u}
\end{align*}\] (29)

where \(-Ke = u_1\), and the filter \(\ddot{u}\) is designed such that
\[\dot{\mu} = -\dot{u} + u_1; \lambda \to \infty\] (30)

where \(\lambda \in [5,50]\) is a positive constant [23,24]. The discussion presented in Secs. 2.3–2.5 about the implications of the non-minimum phase dynamics of the manipulator limit the use of the error signal of the FJM for controller design. The distributed controller proposed these shortcomings, which are presented in the sequel.

### 3.2 The Distributed Higher-Order Differential Feedback Controller

Now, with respect to Eq. (4), where it is defined \(y(t) = \theta(t) + \alpha(t)\), provided that \(y^{(4)}\) exists, then the linearity of the differential operator allows to rewrite that
\[ y^{(4)} = (\theta^{(4)} + \alpha^{(4)}) \] (31)

hence, the following two parallel systems follow from Eqs. (12) and (13):
\[\begin{align*}
\dot{\theta}^{(4)} &= f(\theta) + b_0v_1(t) + d_1(t) \\
\dot{\alpha}^{(4)} &= f(\alpha) + b_0v_2(t) + d_2(t)
\end{align*}\] (32)
\[\begin{align*}
\dot{\theta}^{(3)} &= \dot{\theta}^{(4)} \\
\dot{\alpha}^{(3)} &= \dot{\alpha}^{(4)} - (f(\theta) + b_0v_1(t) + d_1(t))
\end{align*}\] (33)

Then, two separate HODFC systems could be designed for the system, with separate measurements for \(\theta(t)\) and \(\alpha(t)\). Hence, using the measurements of \(\theta(t)\), the following error system is derived:
\[\begin{align*}
e_\theta &= \theta(t) - \theta(t) \\
\vdots \\
\dot{e}_\theta^{(4)} &= \dot{\theta}^{(4)} - \dot{\theta}^{(4)}
\end{align*}\] (35)

and leads to the following equation:
\[\begin{align*}
\dot{e}_{\theta 1} &= e_{\theta 2} \\
\dot{e}_{\theta 2} &= e_{\theta 3} \\
\dot{e}_{\theta 3} &= e_{\theta 4} \\
\dot{e}_{\theta 4} &= \dot{\theta}^{(4)} - \dot{\theta}^{(4)} + (f(\theta) + b_0v_1(t) + d_1(t))
\end{align*}\] (36)

Set in Eq. (36)
\[\begin{align*}
-k_{\theta 1}e_{\theta 1} - k_{\theta 2}e_{\theta 2} - k_{\theta 3}e_{\theta 3} - k_{\theta 4}e_{\theta 4} + \dot{\theta}^{(4)} &= u_{c1} \\
\dot{\theta}^{(4)} - (f(\theta) + b_0v_1(t) + d_1(t)) &= \ddot{u}_{c1}
\end{align*}\] (37)

Or in line with the classical HODFC notation
\[\begin{align*}
\dot{e}_{\theta 1} &= e_{\theta 2} \\
\dot{e}_{\theta 2} &= e_{\theta 3} \\
\dot{e}_{\theta 3} &= e_{\theta 4} \\
\dot{e}_{\theta 4} &= -K_{\theta}e_{\theta 4} + \ddot{u}_{c2}
\end{align*}\] (38)

Similar manipulations based on the \(\alpha(t)\) measurements would admit the second HODFC controller derived in a similar manner, using Eqs. (40)–(42). Hence,
\[\begin{align*}
\dot{\alpha}^{(4)} &= f(\alpha) + b_0v_2(t) + d_2(t) \\
e_{\alpha} &= -\alpha(t)
\end{align*}\] (39)

and to obtain the \(\alpha\)-error dynamics
\[\begin{align*}
\dot{\alpha}_{s1} &= e_{s2} \\
\dot{\alpha}_{s2} &= e_{s3} \\
\dot{\alpha}_{s3} &= e_{s4} \\
\dot{\alpha}_{s4} &= -K_{\alpha}e_{s4} + \ddot{u}_{c2}
\end{align*}\] (41)

where \(u_{c2} = -K_{\alpha}e_{s4}; \dot{\alpha}_{s2} = -\dot{\alpha}_{s2}u_{c2} + u_{c2}; \dot{\lambda} \to \infty\). The implementation of the distributed controller is shown in Fig. 4, such that
\[u(t) = -K_{\theta}e_{\theta} + u_{c1} + \ddot{u}_{c2} - K_{\alpha}e_{\alpha}\] (42)

In the actual implementation, the measurements of \(\theta(t)\) were fed to one HODFC system, while the measurements of \(\alpha(t)\) were fed to the second HODFC.

### 4 Results and Discussion

In order to verify the control performance of the proposed controller, several experiments such as step input, trajectory tracking, and disturbance rejection were conducted. Also, the results obtained from the use of the distributed HODFC were compared with those of the classical HODFC structure used in Ref. [21]. The results from these experiments are presented and discussed in this section.

#### 4.1 Step Input Experiments

Filter in Eq. (42) is not considered in these experiments because of discontinuous input. This discontinuity leads to disturbing the filter behavior. This causes increase in overshoot and the settling time for \(\lambda > 1\). The performances of the controller without filter under step and square wave inputs are shown in Figs. 5 and 6. In Fig. 5, the classical HODF controller tracks a step trajectory with 0.126 s rise time, 0.965 s in settling time, and an overshoot of 28%. Small vibrations persist in the response of the system controlled by the classical HODFC. The distributed controller achieved better control behavior in the given step input, since it eliminates both the overshoots and the joint vibrations quickly. Because of discontinuities of the derivatives of this type of inputs, bounded control signals could not be synthesized for the variable structure controller [6]. Table 1 gives comparison of the controllers based on rise time, overshoot, and settling time.
Note that, among all the trajectories experimented with, the regular jumps in the value of the square-wave trajectory mean that this type of trajectory has the highest derivative content. It is evident that the manipulator controlled by the classical HODFC continuously exhibited overshoot at each falling and rising phases of the square-wave trajectory, hardly reaching steady-state in 0.969 s. Even then, Fig. 6 shows that joint oscillations persist in the response of the system with the classical HODF controller. On the other hand, the system controlled by the distributed controller eliminated the overshoots and settled within 0.683 s, with the joint

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Controller</th>
<th>Rise time (s)</th>
<th>Overshoot (%)</th>
<th>Settling time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step</td>
<td>Classical</td>
<td>0.126</td>
<td>28</td>
<td>0.965</td>
</tr>
<tr>
<td></td>
<td>Distributed</td>
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<td>2.2</td>
<td>0.668</td>
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<td>Square wave</td>
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<td>30</td>
<td>0.969</td>
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<tr>
<td></td>
<td>Distributed</td>
<td>0.23</td>
<td>1</td>
<td>0.683</td>
</tr>
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</table>
oscillations all damped out in the steady-state. The results reported by Kandroodi et al. [22] using fuzzy control in tracking the square-wave trajectory showed that joint vibrations could not be completely damped and steady-state error persists.

4.2 Trajectory Tracking Experiments. Filter in Eq. (42) is applied in these experiments for \( k_1 = k_2 = 5 \). The comparative tracking response of the kane trajectory in the classical and the distributed controllers is shown in Fig. 7. It is evident from the tracking costs shown in Fig. 7 that the distributed HODF controller reduced the tracking error by about 50%. Also, the high-frequency vibrations were eliminated from the steady-state response of the system controlled by the distributed controller. Moreover, the manipulator controlled by the distributed controller tracked the reference trajectory almost immediately, while the system controlled by the classical controller required 2 s to achieve acceptable tracking. The results obtained here are also better than those reported by Kapucu et al. [25] using the systematic command shaping technique and also those of Ref. [6] using the feedback linearization, where in both cases, oscillations persist in the steady-state tracking of a kanelike trajectory. On the other hand, the results compare favorably in accuracy with those reported by Sira-Ramirez et al. [6] using the dynamic variable structure approach. From Fig. 8, similar improvements in tracking performance were obtained with the distributed HODF controller in the tracking of the sine trajectory, when compared with the classical HODF controller. In fact, it could further be observed from Fig. 8 that a lag persists in the response of the manipulator controlled with the classical HODFC. Being that the derivative action is equivalent to some lead compensation, the derivative content in the measurements used for distributed HODFC reduced the tracking lag. It is interesting to note that, at the peak angular positions \((n + 1)\pi/2, n = 0, 2, 4, \ldots\) where the derivative of the sine trajectory reduces to zero, the tracking of the distributed controller degrades to that of the classical controller. By comparison, the distributed controller demonstrated better tracking accuracy of the sine trajectory than that achieved by feedback linearizing

![Classical HOD Input and Output](image1)

![Distributed HOD Input and Output](image2)

**Fig. 7** Kane trajectory tracking performances of the classical and distributed HODFC

![Classical HOD Input and Output](image3)

![Distributed HOD Input and Output](image4)

**Fig. 8** Sine trajectory tracking performances of the classical and distributed HODFC

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Kane</th>
<th>Sine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical HOD</td>
<td>0.006847</td>
<td>0.044727</td>
</tr>
<tr>
<td>Distributed HOD with filter</td>
<td>0.003758</td>
<td>0.024477</td>
</tr>
</tbody>
</table>

**Table 2** Tip error costs of the classical and distributed HODFC
control, reported by Sira-Ramirez et al. [6]. The dynamical variable structure controller presented by Sira-Ramirez et al. [6] achieved consistently accurate steady-state tracking of the sine trajectory, whereas the tracking accuracy of the distributed controller slightly decreased at the points of decreasing derivative of the sine function, especially around the peak values of the trajectory. However, the dynamical variable structure controller suffered from the impact of nonminimum phase dynamics, yielding a substantial initial tracking error. Moreover, the distributed controller achieved tracking within 0.1 s, while the variable structure controller required 4 s to attain its tracking accuracy. It could be expected that an increase in the proportional gain of the controller would compensate for the effect of decreasing derivative signal in sine wave tracking. Trajectory error performance of the controllers is given in Table 2.

4.3 Disturbance Rejection Experiments. The robustness of the two controllers is compared without filter in terms of external disturbance rejection and model uncertainty. For the disturbance rejection experiments, the controller on each system was activated, and the input is set equal to a value of zero. The link was located to an angular position of 30 deg and then suddenly released. It can be seen from the result shown in Fig. 9 that the distributed controller achieved faster and stable disturbance rejection than the classical HODF controller. To test the robustness of the distributed controller to uncertainty of model parameters, the experiments were conducted with a longer link. The results obtained from these experiments are shown in Fig. 10. Based on these results, the distributed HODFC is successful in the uncertainty of model parameters.

5 Conclusions

In this paper, a distributed HODFC is proposed in the trajectory tracking control and vibration control of a FJM. The proposed controller was compared with the classical HODFC and the studies in the literature. Based on the comparison, the distributed HODFC was found successful in eliminating the joint vibrations and reducing trajectory tracking errors. In contrast to the situation generally reported in literature, the distributed HODFC achieves better accurate tracking of the desired discontinuous trajectory, such as the step input and the square-wave trajectories. Moreover, the distributed HODFC as a model-free controller is robust to external disturbance and model uncertainty.

References


