Problem solving and problem posing skills of prospective mathematics teachers about the ‘sets’ subject

Sare ŞENGÜL¹, Yasemin KATRANCI²*

¹Marmara University, Education Faculty Primary Department, Istanbul, Turkey
²Kocaeli University, Education Faculty Primary Department, Kocaeli 41380, Turkey

Abstract

Problem is a topic, event or activity, which no memorized or specified rules are known regarding its solution. Problem posing is a crucial component for mathematics discipline and problem solving is the situation of elimination of confusion in human mind. In this sense, the problem sentence of the study was determined as; ‘What are Problem Solving and Posing Behaviours of Prospective Primary Mathematics Teachers about the ‘Sets’ Subject and Their Experiences and Difficulties Encountered During this Process?’ The purpose of this study is to analyze problem solving and posing behaviours of prospective primary mathematics teachers about the sets subject and to determine the experiences and difficulties that can be seen during these processes. The study was carried out with 56 sophomore students who study at Kocaeli University, Department of Teaching Primary Mathematics. The data was obtained using a data collection form which was prepared by the researchers. The data were analyzed and interpreted in three steps using the content analysis method including i) free problem posing and solving, ii) the experiences that can be seen during problem solving and posing processes, iii) solution recommendations relating to the difficulties (if any) that can be experienced during problem posing and solving processes. It was determined that 53 students could pose problems about the Sets subject suitable to Primary Second Grade level and also they were able to solve these problems. It was seen that 41 students had difficulties in adjusting the degree of difficulty of the problem, not being able to have the full subject knowledge and matching the primary school level during problem posing process. And it was determined that 12 students expressed that they did not experience any difficulties and did not make any solution recommendations. Based on the findings, it was concluded that students could display free problem posing skills but they could not solve the problems according to problem solving steps. This situation can be explained with the fact that the students have not studied problem posing and solving before. It was determined that students had difficulties in matching the primary school level and in adjusting the degree of difficulty of the problem and they believed that they could overcome these difficulties by gaining experience in time. This situation can explained with the fact that since students are sophomore, they do not perceive themselves as real teachers.

Keywords: Sets, problem posing, problem solving

1. Introduction

A problem is a topic, event or activity, which no memorized or specified rules are known regarding its solution (Çanakçı, 2008). According to John Dewey, problem is everything that perplexes, challenges the mind so that it makes belief at all uncertain (Baykul, 2004). Problem posing is a vital component of the mathematics discipline (Stickles, 2006). Problem posing allows students to formulate their own problems, and to use their language skills and knowledge for the problem status (Brown &
Problem posing is a way of analytical thinking (Akay & Noz, 2010), and when previous studies are examined, it can be seen that there are four types of problem posing strategies (Dickerson, 1999; Grundmeier, 2003; Abu-Elwan, 2007): i) free problem posing, ii) semi-structured problem posing, iii) structured problem posing (Akay, 2006) and iv) What if...What if not? (Abu-Elwan, 2007).

In free problem posing strategy; students generate a new problem utilizing a situation from daily life or a given subject. It is more beneficial for students to associate the real-life situations with the mathematics subjects taught, and to pose new problems from these situations. Free problem posing is effective in development of mathematical thinking of students (Akay, 2006). In this study, students were given the subject of “Sets”, and requested to pose problems conforming to the mathematics level of the second grade of Primary Education, utilizing their relevant previous mathematics experiences.

In semi-structured problem posing strategy; students are given an open-ended situation. Then students are requested to pose problems related to this situation, utilizing their knowledge, skills and mathematical experience. Open-ended problems, problems similar to the given problem, problems posed based on pictures provided, and word problems are examples to the semi-structured problem posing strategy (Akay, 2006). He gave students US$ 50 and told them that they could spend the money as they like. Then, he asked to pose a problem as to how they spent the money.

In structured problem posing strategy; the known can be changed to pose a new problem or, the needed can be changed by maintaining the data given; e.g., creation of new problems from solved problems (Akay, 2006). The “What if...What if not?” strategy developed by Brown and Walter belongs to the structured problem posing strategy (Brown & Walter, 1993).

Many researchers studied the relation between problem posing and problem solving (Crespo, 2003; Kar, Özdemir, Ipek & Albayrak, 2010; Nicolau & Philippou, 2004). According to Silver (1994), problem solving that is a component located in the centre of mathematics and mathematical thinking and that is connected with problem posing (Cai & Hwang, 2002; English, 2003) is the situation of elimination of confusion in human mind. According to Altun (2008), problem solving is to know what needs to be done in situations where what needs to be done is not known. Problem solving is, in addition to reaching the result, the situation of finding a way, or relief from challenges (Polya, 1957). In spite of the fact that there is no single solution that can be applied to all problems including mathematical problems, and that each problem requires a different solution, Polya (1957) asserted that there were some specific steps in solving mathematical problems, which are as follows:

1. Understand the problem,
2. Devise a plan,
3. Carry out the plan,
4. Look back (Senemoğlu, 2011).

Kar et al. (2010) found that there is a significant relation between problem posing and problem solving skills of prospective elementary mathematics teachers. With respect to problem solving, it was seen that prospective teachers posing two or more problems had higher level compared to those that pose two or less problems. This study also tries to determine the problem posing and solving skills of prospective elementary mathematics teachers with respect to the Sets subject. In this sense, the following sub-problems were researched:

1. How are the problem-posing skills of prospective teachers?
2. How are the problem-solving skills of prospective teachers?
3. What are the challenges encountered by prospective teachers in the problem posing and solving process?
4. What are the solution strategies recommended by prospective teachers regarding the challenges they encountered (if any)?

2. Method

2.1. Research Design

Qualitative study; a study in which perceptions and events are presented in a realistic and holistic manner in the natural environment, and in which a qualitative process is followed (Yıldırım & Şimşek, 2008). In this respect, this study is a qualitative one, aiming to present results related to a specific situation.

2.2. Working Group

The study was conducted with 56 prospective teachers, who study at the second grade of Kocaeli University, Department of Teaching Primary Mathematics. 47 (83.93 %) of the students that participated in the study were female, and 9 (16.07 %) were male. One (1.78 %) of the students was 18, twenty six (46.43 %) students were 19, twenty two (39.29 %) were 20, five (8.93 %) were 21, one was 22, and one (1.78 %) was 27 years old.
2.3. Data Collection Tools and Collecting Data

Data were obtained using a “data form” prepared by the researchers. This data collection form consisted of three sections: i) Free problem posing and solving related to the “Sets” subject at second grade level of Primary Education; ii) Challenges (if any) encountered during the problem posing and solving process; iii) Solution strategies recommended regarding the challenges (if any) encountered during the problem posing and solving process. The data form also contained demographic variables (sex and age) relating to the students.

Table 1: Data collection form

<table>
<thead>
<tr>
<th>Related to the “Sets” subject:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Pose and solve a problem at the Primary Education second grade level.</td>
</tr>
<tr>
<td>2. Write down the experiences you encountered during the problem posing and solving process (e.g. what kind of challenges did you encounter?).</td>
</tr>
<tr>
<td>3. What are your solution recommendations for the challenges you encountered during the problem posing and solving process?</td>
</tr>
</tbody>
</table>

This data collection form was handed out to students, who were requested to fill it out, by the researchers.

2.4. Data Analysis

Data collected were subjected to descriptive analysis and interpreted at three stages. At the first stage of the data analysis, the free problem posing was analyzed in a way in which the “problem design grading rubric” that had been obtained from Ergun’s (2010) study was adapted to mathematics. The rubric consisted of 6 dimensions, which were comprehensibility of the problem itself (language and expression); consistence of the problem with the mathematical principles; structure of the problem, number of questions asked, type of problem, and solubility of the problem. The maximum score that could be obtained from the first dimension was 20, and it was 18 in the second, 15 in the third, 16 in the fourth, 15 in the fifth and 16 in the sixth dimensions.

Data relating to the problem solving was analyzed and interpreted using the “problem solving grading scale”. The experiences encountered during problem posing and solving process were analyzed and interpreted in the second stage, while the solution recommendations made relating to difficulties associated with the experiences in these processes were analyzed and interpreted in the third. The content analysis aims to gather similar data within the framework of specific concepts and themes, and to arrange and interpret the same in such a way to enable readers to understand (Yıldırım & Şimşek, 2008).

2.5. The Reliability and Validity of Study

In qualitative researches, the validity is to observe the fact being studied in its actual form and in an unbiased manner as far as possible. In order for the subject being studied to be presented as a holistic picture, researcher should confirm the data obtained and results reached through diversification, participant confirmation or colleague confirmation (Yıldırım & Şimşek, 2008). In this context, the data were subjected to analysis by two other researchers, who are specialized in their fields, in addition to the researchers of this study, and the validity of this study was confirmed through colleague confirmation.

Qualitative studies start with the thinking that realities are in a constant change depending on individuals and the environment, and that repetition of the study in similar groups would not allow reaching the same results. In this context, the qualitative researcher should identify the individuals, who serve as the source of data, clearly to ensure reliability (Yıldırım & Şimşek, 2008). In this study, the working group was identified in details to ensure reliability.

3. Findings and Comments

Findings and interpretations related to the first research problem of “How are the problem posing skills of prospective teachers?” are as follows.

Students were asked to pose a problem related to the Sets subject suitable for the second grade of Primary Education. It was seen that 53 out of 56 students that participated in the study posed problems. 3 (5,35 %) students posed problems not related to the Sets subject, and therefore these problems were not included in the evaluation. These 53 problems were evaluated using the “problem posing grading rubric”. The rubric consisted of 6 dimensions, and the problems posed were evaluated based on each sub-dimension.

In this context, in the “comprehensibility of the problem itself” dimension, it was seen that 3 (5,35 %) students obtained 5 points. It means that the text of the problems posed by these three students were partially clear and comprehensible. It was seen that 9 (16,07 %) students obtained 10 points, which means that the texts of the problems they posed were comprehensible, but that their wording was imprecise. It was found that 19 (33,92 %) students obtained 15 points, which means that the texts of the problems they posed were comprehensible, but that they were lacking in terms of mathematical statements. It was seen that 22 (39,28 %) students obtained full points, meaning that the texts of the problems they posed were clear and comprehensible.
In the “consistence of the problem with the mathematical principles” dimension of the rubric, 14 (25 %) students obtained 6 points, which means that the problems they posed were partially consistent with the mathematical principles. 2 (3.7 %) students obtained 12 points. It means that the problems posed were consistent with the mathematical principles, but that they were not consistent with the real life. 18 (66,07 %) students obtained full points in this dimension, which means that the problems they posed were consistent with the mathematical principles.

In the “structure of the problem” dimension, it was seen that 2 (3.57 %) students did not need to use any rules for solving the problems they posed, and therefore these students obtained zero point. It was observed that 34 (60,71 %) students used only 1-2 rules in problems they posed. These students obtained 5 points. 13 (23,21 %) students that obtained 10 points used 3-4 rules in problems they posed. Only 4 (7,14) students obtained full points. It is necessary to know 5-6 rules for solving the problems posed by these students.

In the “number of questions asked” dimension, there were no students who obtained full points. 1 (1,78 %) student asked two questions in the text of the problem and obtained 8 points, while 2 (3,57 %) students asked 3 questions in the text of their problems. 50 (89,28 %) students asked only one question in the problems they posed.

In the “type of problem” dimension, maximum point was 15, but no students could obtain this point. 36 (64,28 %) students designed a problem in the form of simple exercise, and therefore obtained 5 points, while 17 (30,35 %) students designed a problem in the form of normal problem and therefore obtained 10 points.

In the “solubility of the problem” dimension, 47 (83,92 %) students posed problems which were soluble and data given were correct and consistent, and therefore they obtained full point. It was seen that 2 (3,57 %) students posed problems which were not soluble due to extreme complexity of the problem and therefore they obtained 4 points, while 2 (3,57 %) students obtained 8 points due to the fact that the problems they posed were soluble but that the data were incorrect, and 2 (3,57 %) students obtained 12 points due to the fact that the problems they posed were soluble, but lacked some data.

Minimum point that could be obtained from the Rubric was 9, and maximum point was 100. In this context, students that participated in the study obtained points ranging between 33 and 78. 33, 37, 39, 52, 57 and 66 points were obtained by one student each; 56 and 67 points were obtained by two students each; 46 and 58 points were obtained by three students each; 51 and 73 points were obtained by six students each; 68 point was obtained by seven students, 63 point was obtained by eight students; and 78 point was obtained by ten students. Two examples of the problems posed by students can be seen below.

**Problem 1:** Problem posed by the student number 31

![Problem 1](image1.png)

There are 12 people who speak only German and 16 people who speak English in 42 people in a classroom. There are 3 people who speak both English and French. 5 people who speak both English and German, 8 people who speak both German and English. There are 3 students who can speak all languages. French speakers are only two times that only the English speakers. Accordingly, are there how many people who speak only one language in this class?

**Problem 2:** Problem posed by the student number 19

![Problem 2](image2.png)

A and B are two sets. \( s(A) = 2 \times s(B) \) \( s(B - A) = 4 \) and set of “\( A \cap B \)” has 7 subsets. So what is \( s(A) \) set the number of elements?
Findings related to the second research question of “How are the problem-solving skills of prospective teachers?” and respective interpretations are as follows:

It was seen that 53 problems included in the evaluation were soluble. These solutions were analyzed using the “problem solving grading rubric”. This rubric contains four stages relating to solution of the problem. As a result of the evaluations conducted, it was seen that none of the students could obtain any points from this rubric. Even though all 53 problems posed had a solution, it was seen that nothing was done in connection with the dimensions of understanding the problem, devising a plan, carrying out the plan, and looking back to evaluate the solution.

Findings related to the third research question of “What are the challenges encountered by prospective teachers in the problem posing and solving process?” and respective interpretations are as follows:

Students were asked to express the challenges (if any) they encountered during the problem posing process. It was seen that the data obtained in this context were grouped under specific categories, which are “experience, remembering the subject, concretization, level, and experiencing no difficulty”. Data related to these categories are provided in Table 2.

Table 2: Challenges encountered during the problem posing and solving process

<table>
<thead>
<tr>
<th>Challenges Encountered During the Problem Posing and Solving Process</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experience (No previous problem posing and solving experience)</td>
<td>4 (7.54%)</td>
</tr>
<tr>
<td>Failure to remember the subject (failure to remember the Sets subject)</td>
<td>7 (13.21%)</td>
</tr>
<tr>
<td>Concretization (failure to concretize the abstract questions created related to the Sets subject)</td>
<td>4 (%7.54)</td>
</tr>
<tr>
<td>Level (Failure to adjust the suitability of the problems posed to the primary education level)</td>
<td>26 (%49.06)</td>
</tr>
<tr>
<td>Experiencing no difficulty (Having encountered no difficulties)</td>
<td>12 (%22.24%)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>53 (%100)</strong></td>
</tr>
</tbody>
</table>

When the Table 2 is examined, it can be seen that 12 students encountered no difficulties in the process, while 41 students had problems with respect to experience, failure to remember the subject, concretization and level categories.

Data related to the fourth research question of “What are the solution strategies recommended by prospective teachers regarding the challenges they encountered (if any)?” were analyzed and interpreted within the context of the categories related to the challenges encountered. Findings obtained are as follows:

Solution recommendations made for experience category consist of more frequent problem posing and solving exercises, and having good command of the subject.

Solution recommendations made for failure to remember category consist of use of different source books in the problem posing and solving process, and utilization of the technology.

Solution recommendations made for concretization category consist of giving examples from the daily life, and the requirement for further visualization of the subject.

Solution recommendations made for level category consist of review of the primary education mathematics textbooks, defining the problems well, making use of problems posed previously, attaching importance to visualization during the problem-solving process, review of the primary education curriculum well, doing more problem posing and solving exercises, examination of different types of questions, and receiving assistance from colleagues.

4. Conclusion, Discussion and Implications

In this study, which aims to determine the problem posing and solving skills of the prospective primary mathematics teachers related to the Sets subject, it was seen that 94, 64 % of the participants were able to pose and solve problems related to the Sets subject. It was found that the remaining portion of the participants were unable to pose problems related to the subject.

When the problem posing stage is examined, it was observed that approximately half of the students passed the “comprehensibility of the problem” dimension successfully, that the problems posed by 66, 07 % were consistent with the mathematical principles, that 60,71 % posed problems in the form of simple exercises, that 89, 28 % focused on a single question in the problem sentence, and that problems posed by 83,92 % were soluble. Prospective teachers had used simple word problems in the study conducted by Korkmaz and Gür (2006) as well. In this context, this study shows parallelism with the study conducted by Korkmaz and Gür.

None of the students could obtain scores at the problem solving stage. The reason of this situation was that they did not pay attention to the problem solving stages. Therefore, it may be said that, even though they are prospective mathematics teachers, they do not do problem solving exercises.

It was found that, in the problem posing and solving stage, students had the most difficulty in adjusting the level of the problems posed to the level of the primary education. It was followed by “being unable to remember the Sets subject”, in spite of the fact that they were prospective primary mathematics teachers. Other challenges encountered by prospective teachers in this stage were failure to concretize the problems posed, and lack of previous experience in problem posing and solving.

Prospective teachers made recommendations related to the challenges encountered as follows: doing problem posing and solving exercises, examination of different sources in addition to the textbooks, and dealing with different types of questions.
In this context, further studies may be carried out in parallel with this one for the purpose to overcome the challenges indicated and to research the suitability of the solution recommendations.

References


