

Comparative study of performance indices for fundamental robot manipulators

Serdar Kucuk^{a,1}, Zafer Bingul^{b,*}

^a *Kocaeli University, Electronics and Computer Education, Umuttepe Campus, Kocaeli, Turkey*

^b *Kocaeli University, Mechatronics Engineering, Vinsan Campus, Kocaeli, Turkey*

Received 9 October 2004; received in revised form 10 March 2006; accepted 7 April 2006

Available online 5 June 2006

Abstract

In this paper, conventional global and local indices – structural length index, manipulability measure, condition number and Global Conditioning Index (GCI) – have been examined for the workspace optimization of the sixteen fundamental robot manipulators classified by B. Huang and V. Milenkovic [Kinematics of major robot linkages, Robotics International of SME 2 (1983) 16–31]. To find the optimum link length and volumes of these robot manipulators, two design objectives have been used: maximize the workspace area covered by the robot manipulator and maximize the local indices. As an example, a three-link robot manipulator has been studied based on the above design objectives. Also, optimization results of the sixteen robot manipulators have been compared to each other and summarized in tables.

© 2006 Elsevier B.V. All rights reserved.

Keywords: Fundamental robot manipulators; Structural length index; Manipulability measure; Condition number; GCI; Workspace optimization

1. Introduction

The workspace of a robot is a crucial criterion in comparing manipulator geometries. An optimum robotic design provides the largest workspace, the fewest singularities and high stiffness, etc. Reachable and dexterous workspaces mainly characterize the workspace of a robot manipulator. One of the goals of optimal design is to achieve the largest possible volume for the dexterous workspace and the reachable workspace.

There is a close relationship between the kinematics performance and the manipulator structure. This relationship can be conveniently based on workspace capabilities. Therefore, several kinematics-related performance indices have been proposed for serial robot manipulators. The performance indices can be used for comparing the efficiency and usefulness of one serial robot design to another. In serial robot manipulator design, many other important criteria such as cost, payload, accuracy, maximal velocity or stiffness should be taken into account

before making a choice which is related to the type of task that a robot will carry out. In this paper, global (structural length index and GCI) and local (manipulability measure and condition number) performance indices are examined particularly for the workspace optimization of the fundamental robot manipulators. These performance indices are very common metrics in robotics [2,3]. They have also been considered by many others for the purpose of workspace optimization [4–6]. Recently, Pusey et al. [5] used GCI as a performance index of a parallel robot with respect to the force and velocity transmission over the whole workspace. Pashkevich [6] used manipulability and condition number for the geometric synthesis of Orthoglide-type mechanisms.

Several numerical approaches such as heuristic algorithms [7], evolutionary algorithms [8,9], and sequential quadratic programming (SQP) [10,11] have been used for the optimal design of robot manipulators. SQP is the most convenient and fast optimization algorithm in the limited workspace for the comparison of others [12]. Therefore, SQP is preferred for solving multi-objective design optimization problems in this paper.

The organization of the paper is as follows. In Section 2, the two-letter code combination of robot structures and the serial

* Corresponding author. Tel.: +90 262 335 11 48/2212; fax: +90 262 335 28 12.

E-mail addresses: skucuk@kou.edu.tr, serdar_kucuk@yahoo.com (S. Kucuk), zaferb@kou.edu.tr, zbingul@yahoo.com (Z. Bingul).

¹ Tel.: +90 262 303 22 42; fax: +90 262 303 22 03.

Table 1
The joint types for the sixteen robot manipulator structures

Manipulator	S	S	S	C	C	C	N	N	N	R	R	R	R	S	C	N
	S	C	N	S	C	R	S	N	R	C	N	R	S	R	N	C
The first joint	P	P	P	R	R	R	R	R	R	R	R	R	R	P	P	R
The second joint	P	P	R	P	P	P	R	R	R	P	R	P	R	R	R	R
The third joint	P	R	R	P	R	R	P	R	R	R	R	R	P	R	R	P

chain mechanisms considered by Huang and Milenkovic [1] are introduced. In the following section, two performance indices are explained: (i) the manipulability measure and structural length index, and (ii) the condition number and the GCI. The design objectives and optimization method are presented in Section 4. An example of the workspace optimization based on the performance indices is given in Section 5. The results obtained from workspace optimization of robot manipulators are illustrated in Section 6. Finally, conclusions of the study are given in Section 7.

2. Two-letter code description of manipulator structures

Huang and Milenkovic [1] used a two-letter code to classify robot structures. The first letter characterizes the first joint and the first joint's relationship to the second joint. The second letter identifies the third joint and third joint's association to the second joint. The code letters and their meanings are: S is slider, C is rotary parallel to slider, N is rotary perpendicular to rotary and R is rotary perpendicular to rotary or rotary parallel to rotary. The possible combinations of these rotary and prismatic joints comprise the sixteen robot structures: CC, CN, CR, CS, NC, NN, NR, NS, RC, RN, RR, RS, SC, SN, SR and SS. Table 1 gives the joint types for each robot manipulator structure. P and R represent prismatic and revolute joints, respectively.

3. Definition of performance indices

3.1. The manipulability measure and the structural length index

Selection of a robot structure depends on the task to be performed. The task determines structure and position of the robot mechanism. In order to analyze the efficiency of robots, it is needed to have some quantitative measure of their performance. The theory of kinematics synthesis has been considerably furthered during the last decades and various kinematics criteria have been developed to describe the manipulability and dexterity of robot manipulators. Most of these studies were derived from the definition of manipulability. A quantitative measure is required for manipulation in order to compute the capability of a robot manipulator. Yoshikawa [2] proposed kinematics manipulability as a performance measure. When the Jacobian of the manipulator loses its full rank, the manipulator loses one of its degrees of freedom; hence, the manipulability for redundant robot manipulators is defined as

$$w = \sqrt{\det(JJ^T)} \quad (1)$$

where J is the Jacobian of the manipulator. This equation for nonredundant robot manipulators reduces to

$$w = |\det J|. \quad (2)$$

Manipulability measure is very useful for manipulator designing, task planning and fast recovery ability from the escapable singular points for robot manipulators. To obtain a maximally large well-conditioned workspace, the manipulability measure can be used for the design criteria. The objective of a well-designed robot manipulator is to maximize the volume of reachable workspace characterized by high values of manipulability measure. Since the Jacobian is structure dependent, the manipulability measure is a local performance measure and valid at a certain position only. To have a global property of the robot manipulators, the structural length index [13] is used as a global performance index that is based on the ratio of the robot manipulator length sum to the cube root of the workspace volume. A good robotic design is measured by a small length sum with a large workspace volume. The structural length index is given by

$$Q_L = L/\sqrt[3]{V} \quad (3)$$

where V is the volume of reachable workspace and L is the length sum of the robot manipulator given by

$$L = \sum_{i=1}^n (a_{i-1} + d_{i-1}) \quad (4)$$

where a_{i-1} and d_i are the link length and joint offset, respectively.

3.2. Condition number and global conditioning index

The condition number of a matrix is used in numerical analysis to estimate the error generated in the solution of a linear system of equations by the error on the data [14]. Considering in terms of the Jacobian matrix of a robot manipulator, the condition number is an error amplifying factor of actuators, so affecting the accuracy of the Cartesian velocity of the gripper. As a measure of the Jacobian invertibility, condition number is used for planning optimum trajectories and gross motion capability of a robot manipulator in the workspace. The condition number based on the Jacobian matrix is given by

$$\kappa = \|J\| \|J^{-1}\| \quad (5)$$

where $\|\cdot\|$ is the Euclidean norm of the matrix, which is defined as

$$\|J\| = \sqrt{\text{tr}(JNJ^T)} \quad (6)$$

where tr stands for trace and N is a matrix defined by

$$(1/n)[I] \quad (7)$$

where n is the dimension of the square matrix and I is the identity matrix. When the condition number approaches 1 the matrix J is said to be well-conditioned (i.e., is far from singularity) and, in contrast, as the condition number approaches infinity the matrix is said to be ill-conditioned. It is desired for it to be kept as near unity as possible. It ranges from

$$1 \leq \kappa \leq \infty \quad (8)$$

and hence the reciprocal of the condition number, i.e., $1/\kappa$, is defined as the local conditioning index (LCI) to evaluate the control accuracy, dexterity and isotropy of the mechanism [15–17]. LCI is wanted to be as large as possible. Since the Jacobian is structure dependent, its condition number gives a local property of the robot manipulator and it does not demonstrate the general behavior of the robot manipulator precisely. In order to achieve a global property of the manipulator, the following adaptation is used.

$$\eta = \frac{A}{B} \quad (9)$$

$$A = \int_W \left(\frac{1}{\kappa}\right) dW \quad \text{and} \quad B = \int_W dW \quad (10)$$

where W is a specific point of the manipulator workspace, and B is the workspace volume. η stands for GCI, that is the distribution of the condition number of the Jacobian matrix over the entire manipulator workspace, and κ is the condition number at a specific point of the robot manipulator workspace W . The GCI ranges from

$$0 < \eta < 1. \quad (11)$$

As the GCI approaches zero, the workspace has a bad GCI and as the GCI approaches one the workspace has a good GCI. If the boundaries are known, the integration over the workspace can be achieved in Cartesian space. Otherwise, it is better to integrate in joint space, as

$$A = \int_R \left|\left(\frac{1}{\kappa}\right)^2 \Delta\right| d\theta_n \dots d\theta_2 d\theta_1 \quad (12)$$

$$B = \int_R |\Delta| d\theta_n \dots d\theta_2 d\theta_1 \quad (13)$$

where R is the workspace of the robot manipulator in joint space and Δ is the determinant of the Jacobian matrix.

4. Workspace optimization

One of the most complicated problems in manipulator kinematics is to find the optimal geometry. Mathematical equations that describe the behavior of robot kinematics are nonlinear; also they have plenty of terms in general and have rarely known closed-form solutions. Due to the complexity of the optimal design problem, it is not easy

to develop fast prototyping, which allows robot designers to expose structural defects of mechanisms by studying the behavior of their prototypes instead of analyzing troublesome mathematical models. Modern analytical mathematics does not possess generic techniques for having closed-form solutions to nonlinear equations. Hence, iterative methods should be used for solving complicated systems. In this work, a *fminimax* optimization problem solver function from MATLAB is used for manipulator workspace optimization. This optimization function based on a SQP method minimizes the worst-case value of a set of multivariable functions, starting at an initial estimate.

In the multi-objective design optimization problem for a robot's workspace, there are two objectives: (i) the maximum workspace volume covered by the robot manipulator, and (ii) the minimum or maximum local indices. The link lengths are the design variables, which are limited to upper and lower bounds. Based on the robot structure, there are maxima of three optimized design variables or a minimum of one optimized design variable for three link robot manipulators. The consecutive link length ratios were constrained to an upper bound of 2 and to a lower bound of 1.1. The multi-objective design optimization problem is given as follows

maximize or minimize local indices (w, κ)

maximize $V(l_1, l_2, l_3)$ subject to

$$\begin{aligned} G_1 = \frac{l_1}{l_2} &\geq 1.1, & G_2 = \frac{l_1}{l_2} &\leq 2 \\ G_3 = \frac{l_2}{l_3} &\geq 1.1, & G_4 = \frac{l_2}{l_3} &\leq 2 \end{aligned} \quad (14)$$

where the local indices w, κ and workspace volume V are objective functions and G_1, G_2, G_3 and G_4 are nonlinear inequality constraints. Limits of these nonlinear inequality constraints were chosen based on arm lengths of robot manipulators used in industry such as the Yamaha-HXYx series ($l_1 = 1250$ mm, $l_2 = 1050$ mm, $l_3 = 550$ mm), Denso-VM-6083D series ($l_1 = 475$ mm, $l_2 = 385$ mm, $l_3 = 329$ mm), Mitsubishi-RV-2AJ series ($l_1 = 360$ mm, $l_2 = 250$ mm, $l_3 = 160$ mm) and Staubli-Tx40 series ($l_1 = 320$ mm, $l_2 = 225$ mm, $l_3 = 160$ mm). The design variables and l_1, l_2 , and l_3 (link lengths) of fundamental manipulator structures were optimized according to the above multi-objective formulation.

5. Example

The rigid body of the NR (articulated) three-link robot manipulator is given in Fig. 1(a) and the workspace volume is illustrated in Fig. 1(b).

The Jacobian matrix of the NR robot manipulator is given by

$$J = \begin{bmatrix} -l_3 s\theta_1 c\theta_{23} - l_2 s\theta_1 c\theta_2 - d_2 c\theta_1 & -l_3 c\theta_1 s\theta_{23} - l_2 c\theta_1 s\theta_2 & -l_3 c\theta_1 s\theta_{23} \\ l_3 c\theta_1 c\theta_{23} + l_2 c\theta_1 c\theta_2 - d_2 s\theta_1 & -l_3 s\theta_1 s\theta_{23} - l_2 s\theta_1 s\theta_2 & -l_3 s\theta_1 s\theta_{23} \\ 0 & -l_3 c\theta_{23} - l_2 c\theta_2 & -l_3 c\theta_{23} \end{bmatrix}. \quad (15)$$

For the reason of compactness, $(\cos \theta_2 \cos \theta_3 - \sin \theta_2 \sin \theta_3)$, $(\sin \theta_2 \cos \theta_3 + \cos \theta_2 \sin \theta_3)$, $\cos \theta_i$ and $\sin \theta_i$ are abbreviated

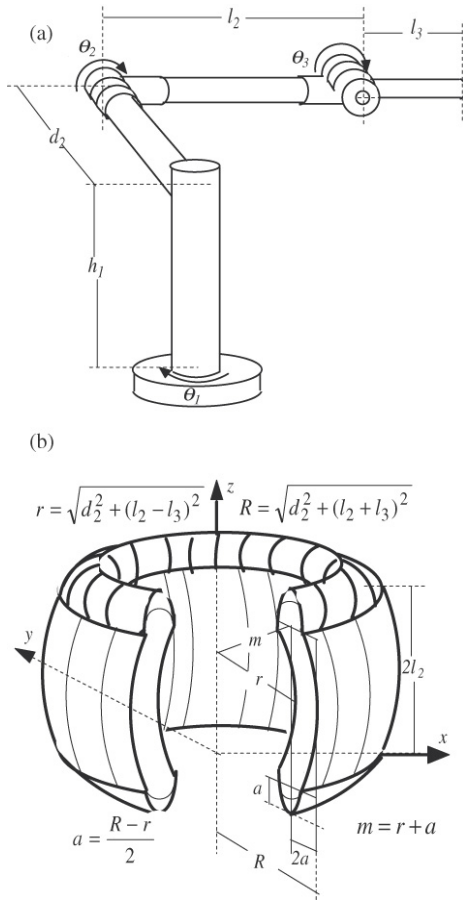


Fig. 1. (a) Coordinate frames, (b) workspace volume of NR robot manipulator.

as $c\theta_{23}$, $s\theta_{23}$, $c\theta_i$, and $s\theta_i$ respectively, where $i = 1, 2, 3$. The manipulability measure and the approximate workspace volume are given as follows

$$w = |l_2 l_3 s\theta_3 (l_3 c\theta_{23} + l_2 c\theta_2)| \tag{16}$$

$$V = 2a^2 \pi^2 m + 8l_2 \pi a (r + a) \tag{17}$$

where $r = \sqrt{d_2^2 + (l_2 - l_3)^2}$, $R = \sqrt{d_2^2 + (l_2 + l_3)^2}$, $a = (R - r)/2$, $m = r + a$. For the NR robot manipulator, the manipulability measure versus the angle of rotation of the second joint, θ_2 , and third joint, θ_3 , is shown in Fig. 2. The NR manipulator can achieve complete gross motion at the highest points of manipulability measure.

The contour analysis in terms of the manipulability measure is shown in Fig. 3. The whole-diagonal ellipses are the best regions in which the NR robot manipulator achieves complete geometric dexterity (like about $\theta_3 = 60^\circ$ and $\theta_2 = 150^\circ$, $\theta_2 = 330^\circ$). At the same time, the curves, which separate the manipulator workspaces, show the singular points.

The NR robot manipulator is examined in terms of the condition number and global conditioning index as follows. The condition number is given by

$$\kappa = \frac{\sqrt{[l_3^2(1 + c^2\theta_{23}) + l_2^2(1 + c^2\theta_2) + d_2^2 + 2l_2l_3(c\theta_3 + c\theta_2c\theta_{23})]e}}{3l_2l_3 \sin \theta_3 (l_3 \cos \theta_{23} + l_2 \cos \theta_2)} \tag{18}$$

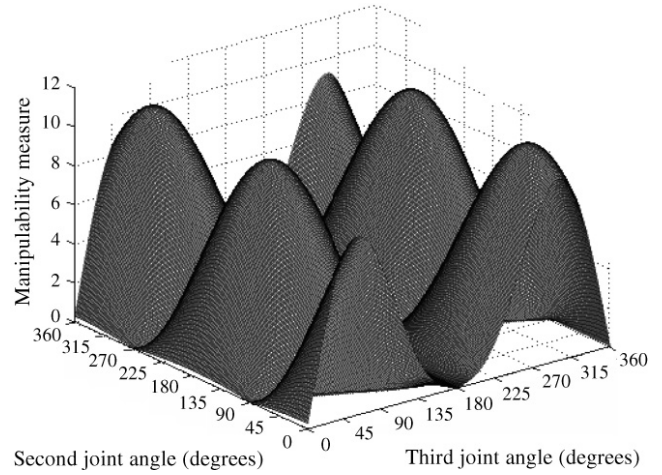


Fig. 2. Manipulability measure of NR robot manipulator versus the angle of rotation of the second joint, θ_2 , and third joint, θ_3 .

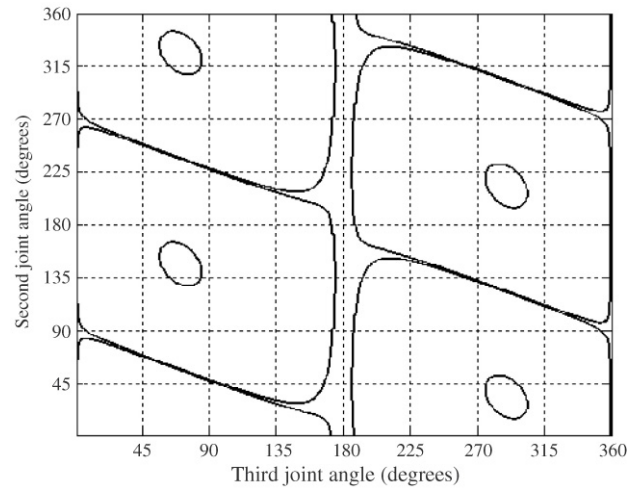


Fig. 3. The contour analysis of the NR robot manipulator in terms of manipulability measure.

where $e = -(2d_2^2l_3^2c^2\theta_{23} + 2l_2^2l_3^2c^2\theta_{23} + 4l_2l_3^3c\theta_2c\theta_{23} + 2l_2^2l_3^3c^2\theta_2c^2\theta_{23} + 2l_2^2l_3^3c\theta_2s\theta_2c\theta_{23}s\theta_{23} + 2l_2l_3^3c\theta_2c^3\theta_{23} + 2l_2^3l_3c^3\theta_2c\theta_{23} + 2d_2^2l_2l_3c\theta_2c\theta_{23} + 3l_2^2l_3^2c^2\theta_2 + 2l_2^3l_3c\theta_2c\theta_{23} + 2l_3^4c^2\theta_{23} + l_2^4c^2\theta_2 + 2l_2^3l_3c^2\theta_2s\theta_2s\theta_{23} + 2l_2l_3^3s\theta_2s\theta_{23}c^2\theta_{23} + l_2^2d_2^2c^2\theta_2)$.

For ease of analysis and comparison, the inverse of the condition number (κ^{-1}) was used. For the NR robot manipulator, κ^{-1} versus the angle of rotation of the second joint, θ_2 , and third joint, θ_3 , is shown in Fig. 4. The NR robot manipulator can achieve complete gross motion at the highest values of κ^{-1} .

A contour analysis of the NR robot manipulator in terms of the condition number is shown in Fig. 5. The whole-vertical hollow ellipses are the best spaces in which the NR robot manipulator has complete geometric dexterity (like about $\theta_3 = 135^\circ$ and $\theta_2 = 120^\circ$, $\theta_2 = 300^\circ$). At the same time, the curves and line ($\theta_3 = 180^\circ$) show the singular points of the NR manipulator.

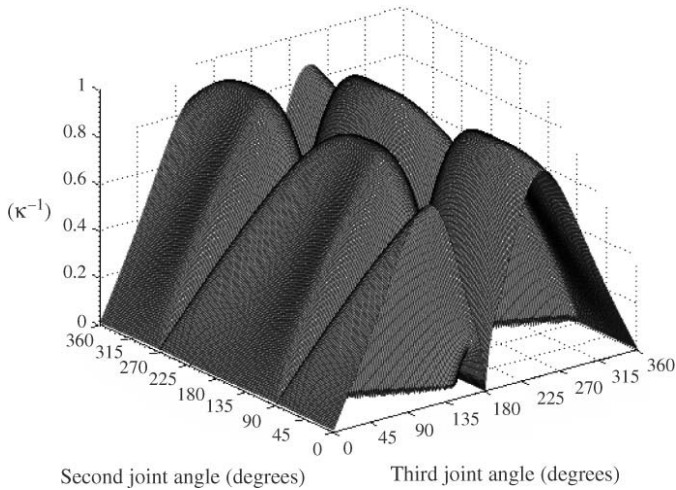


Fig. 4. The inverse of the condition number of the NR robot manipulator versus the angle of rotation of the second joint, θ_2 , and third joint, θ_3 .

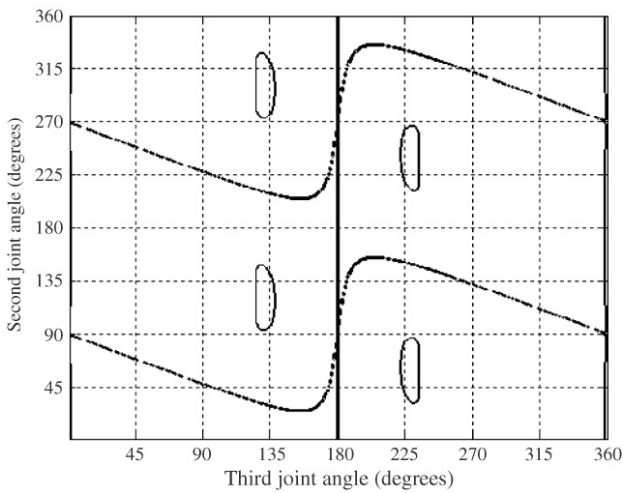


Fig. 5. The contour analysis of the NR robot manipulator in terms of condition number.

6. Optimization results

6.1. Comparison in manipulability measure and structural length index

The structural length index, the manipulability measure and the workspace volume of the robot manipulators are summarized in Table 2. The robot manipulators can be classified into five subgroups with respect to the structural length index values ordered from the lowest to the highest as in Table 2. The Q_L values of robot manipulators vary between 0.8 and 1 in the first group and 1–1.22 in the second group, 1.22–1.32 in the third group, 1.32–2.1 in the fourth group and finally just 3 in the fifth group. The best subgroup includes NN, RR, RN, NR, and CR robot manipulators. Most of the Group I robot manipulators have more revolute joints and spherical workspaces. The RS robot manipulator that is widely used in industry under the name of SCARA in Group III has a higher structural length index. This seems contradictory between the number of robots used in industry and performance index. As

Table 2
 Q_L , w and the workspace volume of the sixteen robot manipulator structures

Group	Manip.	Joint type	Q_L	w	V
I	NN	RRR	0.8201	8.12290	296.830
	RR	RPR	0.8499	9.04300	266.740
	RN	RRR	0.8538	11.4643	258.920
	NR	RRR	0.9053	10.6934	265.430
	CR	RPR	0.9610	4.58220	221.893
II	NS	RPP	1.0470	14.5785	185.140
	RC	RPR	1.1423	6.31100	109.862
	NC	RPP	1.2122	6.64160	91.9440
III	CC	RPR	1.2917	3.63640	91.3930
	RS	RPP	1.2917	3.63640	91.3930
	SR	PRR	1.2917	3.63640	91.3930
	SN	PRR	1.3106	4.91430	87.4910
IV	SC	PPR	1.7382	2.00000	41.1330
	CN	PRR	1.8648	2.00000	33.3090
	CS	RPP	2.0484	2.00000	25.1320
V	SS	PPP	3.0000	–	8.00000

Table 3
The optimized link lengths of the robot manipulators

Manip.	l_1	l_2	l_3	d_1	d_2	d_3
NN	–	1.8182	1.6529	–	2.0000	–
RR	–	2.0000	1.6529	–	1.8182	–
RN	2.0000	1.8182	1.6529	–	–	–
NR	–	2.0000	1.8182	–	2.0000	–
CR	–	2.0000	1.8182	–	2.0000	–
NS	–	2.0000	–	–	2.0000	1.8182
RC	–	2.0000	1.6529	–	1.8182	–
NC	–	1.8182	–	–	2.0000	1.6529
CC	–	2.0000	1.8182	–	2.0000	–
RS	2.0000	1.8182	–	–	–	2.0000
SR	2.0000	2.0000	1.8182	2.0000	–	–
SN	–	2.0000	1.8182	2.0000	2.0000	–
SC	–	2.0000	–	2.0000	2.0000	–
CN	2.0000	2.0000	–	2.0000	–	–
CS	2.0000	–	–	–	2.0000	2.0000
SS	–	–	–	2.0000	2.0000	2.0000

can be seen in Table 2, robot manipulators having prismatic joints have a higher structural length index in general. So, it can be said that the gross motion capability of the robot manipulator is more limited as the number of prismatic joints increases. Moreover, robot manipulators whose first joint is prismatic have a higher structural length index.

The optimized link lengths of the robot manipulators are given in Table 3. As can be seen in Table 3, the optimized link lengths of the prismatic joints are about 2, whereas the other link lengths vary between 1.6529 and 2. Furthermore, the gross motion capability of the robot manipulator in Cartesian space is better as the length sum of the robot manipulator decreases.

6.2. Comparison of condition number and GCI

The GCI, κ^{-1} , and the workspace volume of the sixteen robot manipulators are given in Table 4. The GCI values of the robot manipulators are ordered from the highest to the lowest in

Table 4
The GCI, κ^{-1} and workspace volume of the sixteen robot manipulator structure

Group	Manip.	Joint type	η	κ^{-1}	V
I	NS	RRP	0.2504	0.7367	137.89
	SN	PRR	0.2181	0.3714	87.491
	NR	RRR	0.2178	0.8767	265.43
	NN	RRR	0.2059	0.7132	296.83
II	NC	RRP	0.1851	0.1545	91.944
	RN	RRR	0.1503	0.6269	258.92
	SC	PPR	0.1331	0.8165	41.133
	RR	RPR	0.1227	0.9141	265.75
	RS	RRP	0.1029	1.0000	73.852
III	CC	RPR	0.0816	0.7330	91.391
	SR	PRR	0.0766	1.0000	95.961
	CR	RPR	0.0477	1.0000	221.89
IV	RC	RPR	0.0304	0.7976	15.708
	CS	RPP	0.0249	0.4286	6.2800
	CN	PRR	0.0244	0.4401	29.503
V	SS	PPP	–	–	8

Table 4. Based on the GCI values, the robot manipulators can be classified into five subgroups. The GCI values vary between 0.26 and 0.2 in the first group, 0.2–0.1 in the second group, 0.1–0.04 in the third group, and 0.04–0.02 in the fourth group. Workspace optimization for the SS (Cartesian) manipulator in the fifth group is not necessary to compute since its Jacobian is an identity matrix. Its maximum link lengths were set as 2, and the volume of the workspace was computed based on these values.

The best subgroup includes NS, SN, NR, and NN robot manipulators. As can be seen in Group I, the GCI values of orthogonal robot manipulators (SN, NN, NS) are higher than the others. Thus, the orthogonal robot manipulators are the best for this performance index. On the other hand, the RS and SR robot manipulators in Groups II–III produce the best condition number; however, the GCI values of these manipulators are not good enough comparing with the others. Therefore, it can be concluded that an optimum condition number does not generally produce an optimum GCI.

The optimized link lengths of the robot manipulators are given in Table 5. As can be seen in Table 5, the optimized link lengths of the prismatic joints are generally about 2, whereas the other link lengths vary between 0.5 and 2.

Cartesian manipulators (SS) have poor index values according to Tables 2 and 4. However, they can carry heavy loads and are very convenient for moving parts all over a cluttered site. SCARA robots (RS and SR) are widely used for assembly tasks because they are cheap and feature high velocities and accelerations.

7. Conclusions

In this paper, two different robot workspace variables (manipulability measure and condition number) were studied for the optimal robot design. The robot link parameters were optimized using manipulability measure and condition number.

Table 5
The optimized link lengths of the robot manipulators

Manip.	l_1	l_2	l_3	d_1	d_2	d_3
NS	–	1.8182	–	–	2.0000	1.6529
SN	–	2.0000	1.8182	2.0000	–	–
NR	–	1.8182	1.6529	–	2.0000	–
NN	–	1.8182	1.6529	–	2.0000	–
NC	–	1.8182	–	–	2.0000	1.6529
RN	2.0000	1.8182	1.6529	–	–	–
SC	–	2.0000	–	2.0000	2.0000	–
RR	–	2.0000	1.6529	–	1.8182	–
RS	2.0000	1.4700	–	–	–	2.0000
CC	–	2.0000	1.8182	–	2.0000	–
SR	2.0000	2.0000	1.4970	2.0000	–	–
CR	–	2.0000	1.8182	–	2.0000	–
RC	–	2.0000	0.5000	–	1.0000	–
CS	2.0000	–	–	–	2.0000	1.0000
CN	2.0000	1.8182	–	2.0000	–	–
SS	–	–	–	2.0000	2.0000	2.0000

Also the structures of the robot manipulators were compared based on the structural length index and GCI. Comparison based on the structural length index showed that as the number of revolute joints increases orthogonal robot manipulators and the manipulators with the first two joints parallel have better robot design. Similarly, comparisons based on GCI, orthogonal robot manipulators have better robot design. Considering all results from the tables, NN and NR robot manipulators are the best structures. The optimized link lengths in Tables 3 and 5 may be helpful for designers to obtain optimum manipulator structures. Tables 2 and 4 list the serial robot manipulators according to the workspace variables. This classification may be used by designers when they decide on particular robot designs.

References

- [1] B. Huang, V. Milenkovic, Kinematics of major robot linkages, *Robotics International of SME* 2 (1983) 16–31.
- [2] T. Yoshikawa, Manipulability of robotic mechanism, *The International Journal of Robotics Research* 4 (2) (1985). MIT Press.
- [3] J.K. Salisbury, J.J. Craig, Articulated hands: Force control and kinematics issues, *The International Journal of Robotics Research* 1 (1982) 4–17.
- [4] C. Gosselin, J. Angeles, A global performance index for the kinematics optimization of robotic manipulators, *Journal of Mechanical Design* 113 (September) (1991) 220–223.
- [5] J. Pusey, A. Fattah, S. Agrawal, A. Messina, Design and workspace analysis of a 6-6 cable-suspended parallel robot, *Mechanism and Machine Theory* 39 (2004) 761–778.
- [6] A. Pashkevich, P. Wenger, D. Chablat, Design strategies for the geometric synthesis of orthoglide-type mechanism, *Mechanism and Machine Theory* 40 (2005) 907–930.
- [7] D.C.H. Yang, T.W. Lee, Heuristic combinatorial optimization in the design of manipulator workspace, *IEEE Transactions on Systems, Man and Cybernetics SMC* 14 (4) (1984) 571–580.
- [8] G. Yanga, I.M. Chenb, Task-based optimization of modular robot: Minimized degree-of-freedom approach, *Mechanism and Machine Theory* 35 (2000) 517–540.
- [9] F. Chapelle, P. Bidaud, Evaluation functions synthesis for optimal design of hyper-redundant robotic system, *Mechanism and Machine Theory* (in press).
- [10] M. Ceccarelli, C. Lanni, A multi-objective optimum design of general 3R

manipulators for prescribed workspace limits, *Mechanism and Machine Theory* 39 (2004) 119–132.

- [11] C.D.L. Freudenstein, Optimization of the workspace of a three-revolute open chain manipulators, *International Journal of Robotics Research* 5 (1) (1986) 104–111.
- [12] M.J.D. Powell, A fast algorithm for nonlinearly constrained optimization calculations, in: G.A. Watson (Ed.), *Numerical Analysis*, in: *Lecture Notes in Mathematics*, vol. 630, Springer Verlag, 1978.
- [13] K. Waldron, Design of arms, in: R. Dorf, S. Nof (Eds.), *The International Encyclopedia of Robotics*, John Wiley and Sons, 1988.
- [14] G. Strang, *Linear Algebra and its Applications*, Academic Press, New York, 1976.
- [15] J. Angeles, C. Lopez-Cajun, The dexterity index of serial-type robotic manipulators, in: *ASME Trends and Developments in Mechanisms, Machines and Robotics*, 1988, pp. 79–84.
- [16] G. Alici, An inverse position analysis of five-bar planar parallel manipulators, *Robotica* 20 (2002) 195–201.
- [17] X.J. Liu, J. Wang, G. Pritschow, On the optimal kinematic design of the PRRRP 2-Dof parallel mechanism, *Mechanism and Machine Theory* (in press).



Serdar Kucuk received the degree in electronics and computer education from the University of Marmara, Istanbul, Turkey, in 1995, the M.Sc. degree in electronics and computer education from the University of Marmara, Istanbul, Turkey, in 1998, and the Ph.D. degree in electrical education from the University of Kocaeli, Kocaeli, Turkey, in 2004. Currently, he is working as an Assistant Professor at the University of Kocaeli, in Kocaeli, Turkey.



Zafer Bingul received the B.A. degree from Istanbul Technical University, Istanbul, Turkey and the M.S. and Ph.D. degrees from Vanderbilt University, Nashville, TN, in 1992, 1996, and 2000, respectively, all in electrical engineering. He is currently a Assistant Professor of Mechatronics Engineering, School of Engineering, Kocaeli University, Kocaeli, Turkey. His research interests are robotics and welding automation, optimization, evolutionary algorithms and intelligent control.