Robot Workspace Optimization Based on a Novel Local and Global Performance Indices

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Abstract—In this paper, a novel performance index is introduced for the kinematics optimization of serial robot manipulators. The serial robot manipulators used in order to compare optimization results were classified as in [1]. The new performance index is a combination of a manipulability measure and condition number used by previous authors. To find the optimum link lengths and volumes of these robot manipulators, two design objectives are used: maximize the workspace area and volume of these robot manipulators, and minimize the new local index. As examples, two spherical three-link robot manipulators are examined based on above design objectives. Finally, optimization results of these robot manipulators are obtained and compared to each other.

I. INTRODUCTION

Generally speaking, a robot manipulator structure can be subdivided into a regional structure and orientation structure. The regional structure consists of the arm (first three links), which moves the end-effector to a desired position in the workspace of the robot manipulator. The orientation structure, comprised of the last three links, rotates the end-effector to the desired orientation in the workspace. In this study, the regional structure of the robot manipulators is examined rather than the orientation structure.

The workspace, also called work volume or work envelope of a manipulator, is the volume of space, which the end-effector of the manipulator can reach. The size and shape of the workspace depends on the coordinate geometry of the robot arm, and on the number of degrees of freedom. Some workspaces are quite flat, confined almost entirely to one horizontal plane. Others are cylindrical; still others are spherical. Some workspaces have very complicated shapes. When choosing a robot arm for a certain industrial purpose, it is important that the workspace is large enough to encompass all the points that the robot arm will need to reach, but it is wasteful to use a robot arm with a workspace much larger than necessary.

The workspace of a robot is an important criterion in comparing manipulator geometries. The reachable workspace and the dexterous workspace are two important characteristics used in specifying the workspace of a robot manipulator. The reachable workspace, in which the robot manipulator is able to arbitrarily move its end-effector, does not include any singular points at which the manipulator loses one or more degrees of freedom. The dexterous workspace is the volume of space in which the end-effector can be arbitrarily oriented. The reachable workspace is the volume of space, which the robot can reach in at least one orientation. In the dexterous workspace the robot has complete manipulative capability. The dexterous workspace is a subset of the reachable workspace and is a very important performance index of the robot manipulator. A good robotic design has the volume of reachable workspace as large as possible, and additionally provides a maximum dexterous workspace in this volume.

There is a close relationship between the kinematics performance and design of robot manipulators. Because of this, several kinematics-related criteria have been suggested for designing a well-conditioned robot manipulator that has a dexterous workspace. “Ref. [2]” presented a close relationship between kinematics design and manipulator workspace. Since then, many authors have studied the workspace and singularity analysis of robot manipulators as in [3-9]. A numerical approach to determining the workspace was formulated and solved by tracing boundary surfaces of a workspace as in [10]. They considered the dexterous workspace of manipulators when the manipulator wrist can rotate fully about three axes fully through any point. “Ref. [4]” studied accessible regions of planar manipulators. “Ref. [5], [11] and [12]” developed performance indices that could be used as an optimization and design criteria. “Ref. [13]” defined the determinant of the Jacobian as the manipulability of a robot manipulator and proposed it as a performance criterion. Obviously, the manipulability of a robot manipulator defined in this way is a measure of the manipulator’s inverse Jacobian. “Ref. [14]” showed that the absolute of the Jacobian determinant is not a robust measure of the Jacobian invertibility because the diagonal elements of a square matrix with high numeric values has very large determinant values and its inverse may produce unexpected round off errors. “Ref. [15]” realized this problem and defined a condition number of the Jacobian matrix that yields reasonable numerical values even when the determinant is very small. This performance index has been used extensively by many authors as in [10], [11] and [16]. However, the determinant is still a component of the condition number. This performance index fails at singular points and yields uncontrollable values for the robot manipulator. The new index, introduced here, eliminates the determinant part of the Jacobian matrix completely. Thus, unexpected and uncontrollable values for kinematics optimization of robot manipulators is eliminated.

This paper is written following manner. In Section II, two-letter code combinations and the serial chain mechanisms are introduced. In Section III, the new local and global performance indices are defined. In Section IV, The design
objectives and optimization method used to optimize the reachable workspace volumes and link lengths are presented. In Section V, two spherical three-degree-of-freedom, robot manipulators are given as examples for workspace optimization. The results obtained from the workspace optimization of the sixteen fundamental robot manipulators were discussed in Section VI. Finally the conclusion of this study is given in Section VII.

II. TWO-LETTER CODE DESCRIPTION OF THE ROBOT CONFIGURATIONS

"Ref. [1]" used a two-letter code to classify robot configurations. The first letter characterizes the first joint and the first joint’s relationship to the second joint. The second letter identifies the third joint and third joint’s association to the second joint. The code letters and their meanings are:

S : Slide  
C : Rotary parallel to slide  
N : Rotary perpendicular to rotary, or rotary parallel to rotary

The possible combinations of these rotary and prismatic joints comprise the sixteen robot configurations, which are named as CC, CN, CR, CS, NC, NN, NR, NS, RC, RN, RR, RS, SC, SN, SR and SS.

III. DEFINITION OF PERFORMANCE INDICES

Selection of a robot configuration depends on the task to be performed. The task determines structure and position of the robot mechanism. In order to analyze the efficiency of robots, it is needed to have some quantitative measure of their performance. The theory of kinematic synthesis has considerably furthered during last decades and various kinematic criteria have been developed to describe the manipulability and dexterity of robot manipulators. Most of these studies were derived from the definition of manipulability. A quantitative measure is required for the manipulation in order to compute capability of a robot manipulator. “Ref. [13]” proposed kinematics manipulability as a performance measure. Let $J(\theta)$ be velocity of Jacobian of the manipulator. When $J(\theta)$ loses its full rank, the manipulator loses one of its degrees of freedom; hence, the manipulability is defined as

$$w = \sqrt{\det(J(\theta)J^T(\theta))}$$  \hspace{1cm} (1)

which for nonredundant manipulator reduces to

$$w = \left| \det J(\theta) \right|$$  \hspace{1cm} (2)

where $J$ is Jacobian matrix.

Considering in terms of Jacobian matrix of a robot manipulator, the condition number is an error amplifying factor of actuators, so affecting the accuracy of Cartesian velocity of the gripper. The precision of the control of the robot manipulators depends on the condition number of the Jacobian matrix as in [17]. The condition number based on the Jacobian matrix is given by

$$\kappa = \left\| J \right\| r^{-1}$$  \hspace{1cm} (3)

where $\left\| \cdot \right\|$ is one of the matrix norms of the manipulator Jacobian, given by

$$\left\| J \right\| = \sqrt{\text{tr}(JNJ^T)}$$  \hspace{1cm} (4)

where, $\text{tr}$ stands for trace and $N$ is a matrix defined by

$$1/ n [I]$$  \hspace{1cm} (5)

where $n$ is the dimension of the square matrix and $I$ is the identity matrix.

The new local index for nonredundant manipulator is defined here as the product of manipulability measure and condition number of the Jacobian matrix,

$$\rho_L = w \kappa$$  \hspace{1cm} (6)

where $w$ is given by (2) and $\kappa$ is given by (3). This new performance index is independent of the Jacobian determinant. This can be shown as follows. The condition number can be expressed as,

$$\kappa = \sqrt{m / \Delta}$$  \hspace{1cm} (7)

where $m$ is the numerator value of the product of the Jacobian matrix and its transpose with the inverse Jacobian matrix and its transpose. $\Delta$ is the determinant of the Jacobian matrix of the robot manipulator. If $\rho_L$ is extracted then the following equation is obtained.

$$\rho_L = w \kappa = \Delta \sqrt{m / \Delta} = \sqrt{m}$$  \hspace{1cm} (8)

where $m$ is always a positive value. Because the Jacobian matrix is configuration dependent, $\rho_L$ gives a local property of the robot manipulator. To obtain a global property of the robot manipulator, the following adaptation is made.

$$\rho_G = A^1 / B$$  \hspace{1cm} (9)
where
\[ A = \int_W m \, dW \quad \text{and} \quad B = \int_W dW \]

where, \( \rho_G \) is the new global performance index, \( m \) is the local index at a specific point of the robot manipulator workspace \( W \), and \( B \) is the volume of the workspace.

IV. WORKSPACE OPTIMIZATION

One of the most complicated problem in manipulator kinematics is to find the optimal geometry. Mathematical equations that describe the behavior of robot kinematics are nonlinear, also have plenty of terms in general. The complexity of the optimal design problem urges to develop fast prototyping, which allows robot designers to expose structural defects of mechanism by studying the behavior of their prototypes instead of analyzing troublesome mathematical models. Modern mathematics does not possess generic techniques for having closed-form solutions to nonlinear equations. Hence iterative methods are still used for solving complicated system. In this work, a minimax algorithm which minimizes the worst-case value of a set of multivariable functions, starting at an initial estimate, is used for numerical optimization. This optimization is generally referred to as the minimax problem. It uses a sequential quadratic programming (SQP) method as in [18].

In our multi-objective design optimization problem, there are two objectives: the maximum workspace volume covered by the robot manipulator and the maximize \( \rho_L \). The link lengths are the design variables, which are limited to upper, and lower bounds. Based on the robot configuration, there are a maximum of three optimized design variables or a minimum of one optimized design variable for three link robot manipulators. The consecutive link length ratios were constrained to an upper bound of 2 and to a lower bound of 1.1. The multi-objective design optimization problem is formulated as

\[
\begin{align*}
\text{maximize} & \quad \rho_L (a,b,c) \\
\text{maximize} & \quad V (a,b,c) \\
\text{subject to} & \quad G_i = \frac{a}{b} \geq 1.1, \quad G_2 = \frac{a}{b} \leq 2, \quad G_3 = \frac{b}{c} \geq 1.1, \quad G_4 = \frac{b}{c} \leq 2
\end{align*}
\]

where \( a, b \) and \( c \) (link lengths) are design variables, \( \chi \) and \( V \) are objective functions and \( G_1, G_2, G_3 \) and \( G_4 \) are nonlinear inequality constraints. The link lengths of sixteen different robot manipulators were optimized according to the above multi-objective formulation.

V. EXAMPLES

A. SN Robot Manipulator

The SN robot manipulator in terms of the serial chain mechanisms is drawn in Fig. 1(a). The rigid body whose link parameters assigned according to the D-H method is given in Fig 1(b) as in [19]. The workspace volume is illustrated in Fig 1(c).

The SN (spherical) robot manipulator has two revolute joints (\( \theta_2 \) and \( \theta_3 \)) and one prismatic joint \( (d_1) \). The Jacobian matrix of the SN robot manipulator is given by

\[
J = \begin{bmatrix}
0 & -l_3 \sin \theta_2 \cos \theta_3 - l_2 \cos \theta_2 - l_3 \cos \theta_3 \sin \theta_3 \\
0 & l_3 \cos \theta_3 \\
1 & l_3 \cos \theta_2 \cos \theta_3 - l_2 \sin \theta_2 - l_3 \sin \theta_2 \sin \theta_3
\end{bmatrix}
\]

(11)

The SN robot manipulator is examined in terms of the new local and global indices as follows. The new local index of the SN robot manipulator is

\[
\rho_{SN} = \sqrt{\frac{(1+l_2^2+l_3^2+l_3^2 \cos^2 \theta_3)C}{3}}
\]

(12)

where

\[
C = (l_2^2 + l_3^2) (l_2^2 \cos^2 \theta_2 + \cos^2 \theta_3) + 2l_3^2 \cos^2 \theta_3 \sin^2 \theta_2 + 2l_3^2 \cos \theta_2 \cos \theta_3 \sin \theta_2
\]

The approximate workspace volume covered by SN robot manipulator is

\[
V_{SN} = \frac{4}{3} \pi l_3^3 + (d_1 + 2l_2) \pi l_3^2
\]

(13)

By optimizing the design objectives in (12) and (13), the maximum local index equals 13.7420 and the maximum workspace volume equals 87.491.

For the SN robot manipulator, The new local index versus the angle of rotation of the second joint, \( \theta_2 \) and third joint, \( \theta_3 \) is shown in Fig. 2.
The SN robot manipulator can achieve complete gross motion at the highest values of the new local index.

A contour analysis of the SN robot manipulator in terms of the new local index is shown in Fig. 3. The semi-vertical ellipses and whole-vertical ellipses are the best areas in which the SN robot manipulator has complete geometric dexterity. Also, the angles \( \theta_2 = 90^\circ \), \( \theta_2 = 90^\circ - 270^\circ \) and \( \theta_3 = 270^\circ \) are singular points of the SN robot manipulator.

The new global index for the SN robot manipulator can be calculated according to the definition in (9) by using the optimized \( l_2 \) and \( l_3 \) values in (12) and (13).

\[
\rho_{GSN} = \frac{1}{37.3934} \int_{0}^{2\pi} \int_{0}^{\frac{3\pi}{2}} D \, d\theta_2 \, d\theta_3 \quad (14)
\]

where
\[
D = 930\cos^2\theta_2 + 282\cos^2\theta_3 + 55\sin^2\theta_2 \cos^2\theta_3 \\
+ 61\cos\theta_2 \cos\theta_3 \sin\theta_2 + 112\cos^2\theta_2 \cos^2\theta_3 + 370\cos^4\theta_3 \\
+ 22\cos^2\theta_3 \sin^2\theta_2 + 24\cos^3\theta_3 \cos\theta_2 \sin\theta_2
\]

B. NR Robot Manipulator

The serial chain mechanism, rigid body and workspace volume of NR robot manipulator are illustrated in Fig. 4(a), Fig. 4(b) and Fig. 4(c), respectively.

The NR (spherical) robot manipulator has three revolute joints (\( \theta_1 \), \( \theta_2 \) and \( \theta_3 \)). The Jacobian matrix of the NR robot manipulator is

\[
\mathbf{J} = \begin{bmatrix}
-l_3 \sin \theta_1 \cos \theta_23 - l_2 \sin \theta_1 \cos \theta_2 - d_2 \cos \theta_1 \\
\cdot \\
\cdot
\end{bmatrix} \Rightarrow
\]

\[
\begin{bmatrix}
-l_3 \sin \theta_1 \sin \theta_23 - l_2 \cos \theta_1 \sin \theta_2 \\
-l_3 \cos \theta_1 \sin \theta_23 - l_2 \cos \theta_1 \sin \theta_2 \\
-l_3 \cos \theta_23 - l_2 \cos \theta_2 \\
-l_3 \cos \theta_23 - l_2 \cos \theta_2
\end{bmatrix}
\]

\[ \text{(15)} \]

The new local index of the NR robot manipulator is

\[
\rho_{LNR} = \sqrt{EF/3} \quad (16)
\]

where
\[
E = l_3^2 \cos^2 \theta_23 + 4l_2l_3 \cos \theta_2 \cos \theta_23 + l_2^2 + d_2^2
\]
\[+ l_2^2 \cos^2 \theta_2 + l_2^2 + 2l_2l_3 \sin \theta_2 \sin \theta_23
\]
\[
F = -(2d_2^2 l_2^2 \cos^2 \theta_23 + 2l_2^2 l_3^2 \cos^2 \theta_23 + 4l_2^2 \cos \theta_2 \cos \theta_23 \\
+ 2l_2^2 l_3^2 \cos^2 \theta_2 \cos \theta_23 + 2l_2^2 l_3 \cos \theta_2 \cos \theta_23 \\
+ 2l_2^2 l_3 \cos \theta_2 \sin \theta_2 \cos \theta_23 + 2l_2^2 l_3 \cos \theta_2 \cos \theta_23 \\
+ 2l_2^2 l_3 \cos \theta_2 \sin \theta_2 \sin \theta_23 + 2l_2^2 l_3 \sin \theta_2 \sin \theta_23 \cos \theta_2 \\
+ 2l_2^2 \cos \theta_23 + l_2^2 \cos^2 \theta_2 + l_2^2 d_2^2 \cos^2 \theta_2
\]

where
\[
\cos \theta_23 = \cos \theta_2 \cos \theta_3 - \sin \theta_2 \sin \theta_3 ,
\]
\[
\sin \theta_23 = \sin \theta_2 \cos \theta_3 + \cos \theta_2 \sin \theta_3
\]
The approximate workspace volume is

\[ V_{NR} = 2\pi^2 \pi^2 m + 8/2\pi a(r + a) \] (17)

For the NR robot manipulator, the new the local index versus the angle of rotation of the second joint, \( \theta_2 \) and third joint, \( \theta_3 \) is shown in Fig. 5.

![Figure 5. The new local index of NR robot manipulator versus the angle of rotation of the second joint, \( \theta_2 \) and third joint, \( \theta_3 \).](image)

The contour analysis of the NR robot manipulator in terms of the new local index is shown in Fig. 6.

The new global index for the NR robot manipulator can be calculated as follows:

\[ \rho_{G_{NR}} = \frac{1}{265.43} \int \int \left( \frac{\rho}{L} \right) d\theta_2 d\theta_3 \] (18)

where

\[ G = 2.73\cos^2 \theta_2 + 12\cos \theta_2 \cos \theta_2 + 3.3\cos^2 \theta_2 + 6\sin \theta_2 \sin \theta_3 + 10 \]

The contour analysis of the NR robot manipulator is shown in Fig. 6.

![Figure 6. The contour analysis of NR robot manipulator.](image)

VI. OPTIMIZATION RESULTS

The new local and global performance indices and the workspace volume of the robot manipulators are given in Table II (M, J. T., R, P and \( V \) represent manipulator, joint type, revolute joint, prismatic joint and workspace volume of robot manipulators, respectively). The new global performance index of the sixteen manipulators is ordered from the highest value to the lowest value in Table II. The spherical robot manipulators (SN, RN, NR and NN) have better new global performance index values than the others. The RR and RC robot manipulators have high local index values, but the global performance index of these manipulators is not high compared to their local index values. A high local performance index does not necessarily produce a high global performance index.

<table>
<thead>
<tr>
<th>M.</th>
<th>J. T.</th>
<th>( \rho_G )</th>
<th>( \rho_L )</th>
<th>( V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN</td>
<td>PRR</td>
<td>3.9476</td>
<td>13.742</td>
<td>87.491</td>
</tr>
<tr>
<td>RN</td>
<td>RRR</td>
<td>1.3203</td>
<td>24.083</td>
<td>258.92</td>
</tr>
<tr>
<td>NR</td>
<td>RRR</td>
<td>1.3144</td>
<td>25.855</td>
<td>265.43</td>
</tr>
<tr>
<td>N</td>
<td>RRR</td>
<td>1.0136</td>
<td>17.633</td>
<td>296.83</td>
</tr>
<tr>
<td>CN</td>
<td>RRR</td>
<td>0.9717</td>
<td>6.3700</td>
<td>29.503</td>
</tr>
<tr>
<td>NC</td>
<td>RRP</td>
<td>0.7863</td>
<td>16.009</td>
<td>91.944</td>
</tr>
<tr>
<td>NS</td>
<td>RRR</td>
<td>0.5977</td>
<td>22.922</td>
<td>137.89</td>
</tr>
<tr>
<td>CC</td>
<td>RPR</td>
<td>0.4230</td>
<td>9.6440</td>
<td>91.391</td>
</tr>
<tr>
<td>CR</td>
<td>RPR</td>
<td>0.7373</td>
<td>6.7420</td>
<td>221.89</td>
</tr>
<tr>
<td>SC</td>
<td>PPR</td>
<td>0.3284</td>
<td>2.4490</td>
<td>41.113</td>
</tr>
<tr>
<td>SR</td>
<td>PPR</td>
<td>0.2516</td>
<td>6.4580</td>
<td>95.961</td>
</tr>
<tr>
<td>RS</td>
<td>RPR</td>
<td>0.2477</td>
<td>6.4580</td>
<td>91.391</td>
</tr>
<tr>
<td>RR</td>
<td>RPR</td>
<td>0.1745</td>
<td>20.668</td>
<td>266.74</td>
</tr>
<tr>
<td>RC</td>
<td>RPR</td>
<td>0.0359</td>
<td>12.284</td>
<td>109.86</td>
</tr>
<tr>
<td>SS</td>
<td>PPP</td>
<td>-</td>
<td>-</td>
<td>8.0000</td>
</tr>
</tbody>
</table>

The optimized link lengths of the robot manipulators are given in Table III. As can be seen in Table III, the optimized link lengths of the prismatic and revolute joints vary between 1.6529 and 2. Furthermore, the gross motion capability of the robot manipulator in Cartesian space is better as the number of revolute joints increase.

<table>
<thead>
<tr>
<th>M.</th>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>( l_3 )</th>
<th>( d_1 )</th>
<th>( d_2 )</th>
<th>( d_3 )</th>
</tr>
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<tbody>
<tr>
<td>SN</td>
<td>-</td>
<td>2.0000</td>
<td>1.8182</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RN</td>
<td>2</td>
<td>1.8182</td>
<td>1.6529</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>NR</td>
<td>-</td>
<td>1.8182</td>
<td>1.6529</td>
<td>2</td>
<td>2.0000</td>
<td>-</td>
</tr>
<tr>
<td>N</td>
<td>-</td>
<td>1.8182</td>
<td>1.6529</td>
<td>2</td>
<td>2.0000</td>
<td>-</td>
</tr>
<tr>
<td>CN</td>
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<td>1.8182</td>
<td>-</td>
<td>2</td>
<td>2.0000</td>
<td>-</td>
</tr>
<tr>
<td>NC</td>
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<td>1.8182</td>
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<td>-</td>
<td>2.0000</td>
<td>1.6529</td>
</tr>
<tr>
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<td>1.8182</td>
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<td>-</td>
<td>2.0000</td>
<td>1.6529</td>
</tr>
<tr>
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<td>1.8182</td>
<td>-</td>
<td>2.0000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CR</td>
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<td>1.8182</td>
<td>-</td>
<td>2.0000</td>
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<td>-</td>
<td>2.0000</td>
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</tr>
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<td>-</td>
<td>2.0000</td>
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</tr>
<tr>
<td>RS</td>
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<td>1.8182</td>
<td>-</td>
<td>2.0000</td>
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<tr>
<td>SS</td>
<td>-</td>
<td>-</td>
<td>2.0000</td>
<td>2.0000</td>
<td>2.0000</td>
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</tr>
</tbody>
</table>

VIII. CONCLUSION

The sixteen fundamental robot manipulators were compared based on the new local and global performance indices. The comparison showed that spherical robot manipulators have higher global performance indices. Thus, these robot manipulators are better robot designs and achieve gross motion in all three spatial dimensions. On the other
hand, the new local performance indices of some spherical robot manipulators are smaller than the new local indices of other robot configurations. This demonstrates that the high local performance index does not always produce high global performance index. The new local index cannot be used for comparing the robot manipulators with each other as a global performance index. It just shows the gross motion ability of the robot manipulator itself.

REFERENCES