Effect of Reinforcement Flexibility on Compressive Behavior of Strip-Shaped Elastomeric Bearings

S. Pinarbasi¹, U. Akyuz², Y. Mengi³

¹Department of Civil Engineering, Kocaeli University, Turkey
²Department of Civil Engineering, METU, Turkey
³Department of Engineering Sciences, METU, Turkey

Abstract

Composed of several rubber layers bonded to reinforcing sheets, elastomeric bearings have widely been used for isolation of structures from vibrations. In an attempt to develop light-weight and low-cost isolators to be used in developing countries, recent studies have proposed to replace steel reinforcement with fiber reinforcement. There are three parameters characterizing the behavior of a fiber-reinforced bearing: “shape factor” of the bearing, Poisson’s ratio of the elastomeric material and flexibility of the reinforcing sheets. This study aims to investigate the effect of reinforcement flexibility on compressive behavior of elastomeric bearings with different geometrical and material properties by using the advanced analytical solutions derived by Pinarbasi and Mengi (2008). Free from two of the three basic assumptions of the commonly-used pressure method, i.e., the “pressure” and the “plane sections remain plane” assumptions, these analytical solutions are valuable tools for a detailed study on compressive behavior of rubber bearings reinforced with extensible reinforcements. Analyses show that the effect of reinforcement flexibility on compressive behavior of a fiber-reinforced bearing highly depends on its geometric and material properties. While the bearings with low shape factors is not affected from reinforcement flexibility considerably unless the reinforcement stiffness is too low, the bearings with high shape factors can be very sensitive to the reinforcement flexibility especially if they are composed of nearly incompressible.

Keywords: seismic isolation, rubber, fiber reinforced bearing, compression, flexible reinforcement.
1 Introduction

Composed of several rubber layers bonded to reinforcing sheets, elastomeric bearings are widely used for isolation of structures from vibrations. While, in most of the earlier applications, these rubber-based isolators are reinforced with steel shim plates, recent studies (e.g., Kelly, 2002; Tsai, 2004; Moon et al., 2002) propose replacement of steel reinforcement with fiber reinforcement. With this replacement, it is possible to produce both lightweight and cost-effective isolators to be used in developing countries and/or in low-cost housing.

Analytical studies on elastomeric bearings have shown that the behavior of a multi-layered elastomeric bearing under pure compression can be deduced from the behavior of a typical interior elastomer layer bonded to reinforcing sheets at its top and bottom faces (Kelly, 1997). Earlier studies on steel-reinforced elastic layers (e.g., Gent and Lindley, 1959; Gent and Meinecke, 1970; Chalhoub and Kelly, 1991; Tsai and Lee, 1998; Pinarbasi et al., 2006) have clearly showed that the compressive stiffness of a bonded elastic layer can be much larger than its unbonded stiffness. These studies have also indicated that the behavior of a steel-reinforced elastic layer under compression is mainly controlled by the aspect ratio of the layer and compressibility of the elastic material. Considering that the flexibility of the reinforcements may also affect the behavior of a bonded elastic layer in case of fiber-reinforcement, it can be concluded that there are three parameters characterizing the behavior of an elastic layer bonded to flexible reinforcements: “shape factor” of the layer (S), i.e., the ratio of one loaded area to bulge free areas for an interior rubber-reinforcement unit, Poisson’s ratio of the material (ν) and flexibility of the reinforcing sheets (kf). Investigation of the effects of each parameter on layer behavior is essential for thorough knowledge on behavior of fiber-reinforced elastomeric bearings.

Recently, elastic layers bonded to flexible reinforcements have been analyzed by using an approximate theory which is based on modified Galerkin method (Pinarbasi and Mengi, 2008). Eliminating the well known “pressure” and “plane sections remain plane” assumptions (refer, to Kelly (1997) for details), the theory leads to advanced closed-form solutions, which can be used as a valuable tool for a detailed study on compressive behavior of elastomeric bearings with flexible reinforcement. This paper mainly aims to study the effect of reinforcement flexibility on compressive behavior of elastomeric bearings using the analytical solutions derived from the above-mentioned theory for “infinite-strip” shaped bonded elastic layers. Since the effect of reinforcement extensibility also depends on the geometrical and material properties of the layer itself, the discussions inherently include a study on the effects of the other two parameters, S and ν, on behavior of fiber-reinforced bearings.

2 Closed-Form Expressions for Compressive Behavior of Elastic Layers Bonded to Flexible Reinforcements

Figure 1a shows the undeformed configuration for a rectangular elastic layer of uniform thickness $t$ bonded to flexible reinforcements with equivalent thickness $t_f$. 

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at its top and bottom faces. The length of the layer is much larger than its width $2w$ and thickness $t$. The layer can be defined referring to a two-dimensional rectangular coordinate system $(x_1, x_2)$ with its origin located at the center of the layer since the layer is in a state of plane strain. If the layer is uniformly compressed by applying a concentric axial load of $P$, it will deform as shown in Figure 1b.

![Figure 1. Undeformed and deformed configurations for an elastic layer bonded to flexible reinforcements](image)

This compression problem has recently been solved by Pinarbasi and Mengi (2008) using an approximate theory based on a modified version of Galerkin method (Mengi, 1980) and obtained advanced closed-form solutions for displacement/stress distributions and effective compression modulus. The following expressions for the displacement components ($u_1$ and $u_2$) and for the compression modulus ($E_c$) are directly taken from Pinarbasi and Mengi (2008).

\[
\begin{align*}
  u_1 &= \left\{ \frac{3 \Delta \lambda}{2t \alpha \beta_1 \cosh(\beta_1 w)} \sinh(\beta_1 x_1) \left(1 - \frac{4x_1^2}{t^2}\right) \right. \\
  &+ \frac{\Delta \lambda}{t \alpha \beta_1} \left(x_1 - \frac{\sinh(\beta_1 x_1)}{\beta_1 \cosh(\beta_1 w)}\right) \\
  &+ \frac{\Delta \lambda}{t \alpha \beta_1^2} \left(x_1 - \frac{\sinh(\beta_1 x_1)}{\beta_1 \cosh(\beta_1 w)}\right) \\
  \end{align*}
\]

\[
\begin{align*}
  u_2 &= \frac{30 \Delta \lambda}{t \alpha \beta_1 \cosh(\beta_1 w)} \\
  &\cdot \left[ \frac{1}{\beta_1^2 - \beta_{21}^2 \sinh(\beta_{21} w)} \cosh(\beta_{21} x_1) \right] - \frac{\Delta x_2}{t} \\
  &+ \frac{\mu + \lambda}{\mu} \frac{1}{\beta_1^2 - \beta_{21}^2} \cosh(\beta_1 x_1) \\
  &+ \frac{\mu + \lambda}{\mu} \frac{1}{\beta_1^2 - \beta_{21}^2} \cosh(\beta_1 w) \\
  \end{align*}
\]
\[ E_c = \alpha - \frac{\lambda^2 \beta_{10}^2 \tanh(\beta_{11} \nu)}{\alpha \beta_{10}^2} - \frac{\lambda^2 \beta_{11}^2}{\alpha \beta_{11}^2} \]  \tag{3}

where
\[ \beta_{10}^2 = \frac{12 \mu}{\alpha t}, \quad \beta_{11}^2 = \frac{12 \mu}{k_f t}, \quad \beta_1^2 = \beta_{10}^2 + \beta_{11}^2, \quad \beta_{21}^2 = \frac{60 \alpha}{\mu t^2} \]  \tag{4}

It is to be noted that in Eqs. (1-4), \( \alpha = 2\mu + \lambda \) where \( \mu \) and \( \lambda \) are Lamé’s constants, in-plane stiffness of the reinforcing sheets \( k_f = E_f t_f / (1 - \nu_f^2) \) where \( E_f \) and \( \nu_f \) are, respectively, elasticity modulus and Poisson’s ratio of the reinforcing sheets, and \( \Delta \) is the applied vertical displacement. Using the displacement-stress relationships of linear elasticity, stress distributions can easily be obtained from Eqs. (1,2). Due to their lengthy forms, they are not presented here.

3 Effect of Reinforcement Flexibility on Compressive Behavior

For a bonded elastic layer, there are two limiting cases as far as the flexibility of the reinforcing sheets to which the layer is bonded at its top and bottom faces is concerned: the layer behavior approaches its “unbonded (no-reinforcement)” behavior when the stiffness of the reinforcing sheets tends to zero and approaches its “rigid-reinforcement” behavior when their stiffness tends to infinity. This section aims to investigate the effect of reinforcement flexibility on compressive behavior of bonded elastic layers with different geometrical and material properties.

Figure 2 shows the variation of compression modulus \( E_c \) with Poisson’s ratio for various shape factors and two specific values of “stiffness ratio” \( k_f^* = k_f / (\mu t) \): 30000 and 300, corresponding, respectively, to a considerably high and a relatively low stiffness ratio. It is worth noting that the value of 30000 for \( k_f^* \), which can be considered as a typical value for a fiber-reinforced rubber bearing, is calculated using the typical values \( (E_f = 210 \text{ GPa}, \nu_f = 0.3, t_f = 0.27 \text{ mm}, t = 3 \text{ mm}, \mu = 0.7 \text{ MPa}) \) quoted in literature (see, e.g., Kelly, 2002). In order to see the direct effect of reinforcement flexibility on compression modulus, \( E_c \) values in Figure 2 are normalized by those computed ignoring the flexibility of the reinforcements, denoted as \( E_{c, \text{rigid}} \).

The graphs clearly show how the effect of \( k_f^* \) on \( E_c \) depends on the particular values of \( S \) and \( \nu \). \( E_c \) of high shape factor (HSF) layers decreases significantly when \( k_f^* \) decreases from 30000 to 300 if \( \nu \) is close to 0.5. For example, for \( S = 30 \), the modulus ratio is 0.44 when \( \nu = 0.499 \) and is as low as 0.07 when \( \nu = 0.5 \). For the same value of \( k_f^* \), the effect of reinforcement flexibility is much less in low shape factor (LSF) layers. For instance, the layer with \( S = 1 \) does not “sense” the flexibility of the reinforcements if \( k_f^* = 300 \) even when \( \nu = 0.5 \). It can also be seen that when \( k_f^* = 30000 \), even HSF layers behave as if they were rigidly-bonded provided that their shape factors are not too large and/or \( \nu \leq 0.499 \).
Figure 2. Effect of reinforcement flexibility on compression modulus

Figure 3. Effect of reinforcement flexibility on axial stress distribution in lateral direction for $S=1$ and $S=30$
From Figure 2, it can also be concluded that a bonded elastic layer attains its “incompressible” modulus at a smaller value of \( \nu \) if it has a smaller \( S \) and/or \( k'_f \).

The graphs in Figure 3 show the distribution of axial stress \( (\tau_{22}) \) over the central plane \((x_3=0)\) of the layer, where it attains its maximum values, for two different \( S \) values; 1 and 30, representing, respectively, LSF and HSF layers and two different \( k'_f \) values; 30000 and 30, the smaller of which is deliberately selected to be so low since, the LSF layer is observed not to be influenced from \( k'_f \) even when \( k'_f = 300 \) (Figure 2). In the graphs, stress values are normalized with respect to the uniform pressure, i.e. \( E_c \varepsilon_c \), where \( \varepsilon_c = \Delta / t \). Figure 3 shows strong effect of reinforcement flexibility on HSF layers with nearly/strictly incompressible materials. When \( k'_f \) decreases, maximum axial stress, which occurs at \((x_1=0, x_3=0)\), decreases significantly and parabolic stress distributions become much more uniform. Thus, the effect of \( k'_f \) on compressive behavior of a bonded elastic layer is exactly the same as that of \( \nu \). On the other hand, stress distributions are almost insensitive to \( k'_f \) even when \( k'_f \) is as low as 30 if \( S \) is small and/or \( \nu < < 0.5 \).

From the graphs in Figure 4, which plot normal stress \( (\tau_{11} \text{ and } \tau_{22}) \) distributions in axial direction for \( S=1 \) when \( k'_f = 30 \), one can see that, the “pressure assumption” is not valid for LSF layers bonded to flexible reinforcements since normal stresses are neither constant nor equal to each other through the layer thickness. The distributions are highly nonuniform especially if the layer material is nearly/strictly incompressible. Our studies show that a similar conclusion is valid for HSF layers if the reinforcement stiffness is low. While normal stresses in an HSF layer are constant through the layer thickness even when \( k'_f \) is considerably low, as shown in Figure 5, axial stress distributions start to deviate from lateral stress distributions as \( k'_f \) decreases. From Figure 4, it can also be observed that the presence of slight compressibility (e.g., \( \nu = 0.499 \)) does not affect the behavior of LSF layer. This is due to the fact that, as mentioned earlier, LSF layers reach their incompressible behavior at much smaller values of \( \nu \) than HSF layers.

Figure 5 illustrates significant effect of \( k'_f \) on lateral normal stress distribution through the layer thickness of an HSF layer. Stress values decrease considerably when \( k'_f \) decreases if the material is strictly/nearly incompressible. The effect of the slight compressibility (e.g., \( \nu = 0.499 \)) on the layer behavior when \( k'_f = 30000 \) is also worth mentioning here. Unlike an LSF layer, the behavior of an HSF layer can be very sensitive to the slight compressibility in the layer material if the stiffness of reinforcing sheets is large (Figure 5a). As shown in Figure 5b, this sensitivity disappears as the flexibility of the reinforcing sheets increases and HSF layers start to behave like LSF layers.

One of the most important design parameters for elastomeric bearings is the maximum shear stress/strain developed in the elastomer layers under uniform compression. Figure 6 shows the effect of reinforcement flexibility on maximum shear stress in a bonded elastic layer. Parallel with the earlier conclusions, reinforcement flexibility is effective only when \( S \) is sufficiently large and \( \nu \) is sufficiently close to 0.5. One should also note how the “normalized” maximum shear stress values increase as \( S, \nu \) or \( k'_f \) decreases.
Figure 4. Normal stress distributions in axial direction for $S=1$ when $k_f/\mu t = 30$

Figure 5. Effect of reinforcement flexibility on lateral normal stress distribution in axial direction for $S=30$

Figure 6. Effect of reinforcement flexibility on maximum shear stress in a bonded strip-shaped layer under uniform compression
4 Conclusions

Bearings composed of several elastomeric layers bonded to reinforcing sheets are widely used in many engineering applications. While, in most of the earlier applications, as reinforcing agents, steel shim plates have been used in these bearings, recent studies propose flexible fiber reinforcement. This study investigates the effect of reinforcement flexibility on behavior of elastomeric bearings under uniform compression.

Analyses show that the effect of reinforcement flexibility on compressive behavior of a fiber-reinforced bearing highly depends on its geometric and material properties. While the bearings with low shape factors is not affected from reinforcement flexibility significantly unless the reinforcement stiffness is too low, the bearings with high shape factors can be very sensitive to the reinforcement flexibility especially if they are composed of nearly/strictly incompressible.

References