Non-constancy of Speed of Light in Vacuum for Different Galilean Reference Systems and Momentum and Energy of a Particle

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Abstract—The conservation of energy and momentum of a particle is investigated under a ‘modified Lorentz transformation’ which is a transformation law that emerges as compulsory after considering two lossy media to which inertial frames $K$ and $K'$ are attached. This emergence is an outcome of the failure of Special Relativity Theory to account for the mentioned loss in one of the media, as proved by the author in reported preceding work. This ‘modified Lorentz transformation’ incorporates different speeds of light in vacuum $c$ and $c'$ for $K$ and $K'$. The velocity addition law under this transformation is given. Dependence of the relativistic expressions of momentum and energy for a particle which are relegated to their nonrelativistic values in the limiting case, on the magnitude of velocity is sought for. To this end collision and scattering of two identical particles are considered. A small scattering angle $\theta'$ is assumed in a glancing collision of the two particles, and the demanded dependences on the magnitude of the velocity of the particle are derived. This is achieved by considering a Taylor expansion of the conservation of energy equation around the point $\theta' = 0$. The results dictate a covariant but not invariant relation between the energy of a particle and its mass. This is due to the assumption of existence of different speeds of light in vacuum $c$ and $c'$ for frames $K$ and $K'$. This development is identical with an existing one in the literature, but here the principle of constancy of speed of light is put aside on the basis of the falsity of Special Relativity Theory that was established previously.

1. INTRODUCTION

Consider two Galilean reference frames $K$ and $K'$, of which $K'$ is in uniform rectilinear motion with speed $v_1$ with respect to $K$.

The three assumptions of Special Relativity Theory are [1]

i) The principle of relativity (i.e., that laws of physics are the same in all Galilean reference systems, there exists no preferred Galilean system),

ii) Assumption of homogeneity of space-time (to infer linearity of transform equations),

iii) Assumption of isotropy of space (to infer reciprocity also using the principle of relativity).

The author has established in [2, 3] that the principle of constancy of speed of light in vacuum [4–6] that is a consequence of the above three assumptions, is false in the general case, by considering an electromagnetic system, wherein the above inertial frame $K$ is attached to a medium which is simple but lossy, whereas inertial frame $K'$ is attached to a medium which is a perfect electric conductor filling the half space, such that the interface of the two media is an infinite plane perpendicular to the velocity of $K'$ in direction of $Ox$ axis with respect to $K$.

It is noted in [3] as the outcome of this finding that if $c \neq c'$ (inequality of speeds of light in vacuum for $K$ and $K'$) is assumed a ‘modified Lorentz transformation’ has to be derived that considers unequal $c$ and $c'$. Derivation of this transformation is given in [2]. Using the results of this transform the following law of addition of velocities can be derived.

$$
\begin{align*}
    u'_x &= \frac{u_x' - v_2}{c'/c - ru'_x/c} & & u'_y = \frac{u_y'}{\alpha(c'/c - ru'_x/c)} & & u'_z = \frac{u_z'}{\alpha(c'/c - ru'_x/c)}
\end{align*}
$$

Here $u_x, u_y, u_z$ and $u'_x, u'_y, u'_z$ are respectively the velocity components of a moving object as observed from $K$ and $K'$. Here $\alpha = (1 - r^2)^{-1/2}$ with $r = -v_1/c = v_2/c'$ and $v_2$ being the speed of $K$ with respect to $K'$.

2. FUNCTIONAL FORM OF MOMENTUM AND ENERGY OF A PARTICLE

This section is exclusively based on [6] and only the changes called for by the non-constancy of speed of light in vacuum for different Galilean reference systems are implemented in the derivation of functional form of momentum and energy of a particle.
For a particle with speed small compared to speed of light in vacuum

\[ \vec{p} = m \vec{u} \]  
\[ E = E(0) + \frac{1}{2} m u^2 \]

(1a) \hspace{1cm} (1b)

\( m \) is the mass of the particle, \( \vec{u} \) is its velocity and \( E(0) \) is a constant corresponding to the rest energy of the particle. We shall attempt to derive the momentum and energy of a particle in line with the ‘modified Lorentz transformation’ law of velocities. The modified Lorentz transformation is reported in [2, 3]. As stated above this transformation deviates from the Lorentz transformation in that it incorporates two different speeds of light for systems \( K \) and \( K' \).

The only possible general versions of (1) in line with the principle of relativity are:

\[ \vec{p} = M(u) \vec{u} \]  
\[ E = E(u) \]

(2a) \hspace{1cm} (2b)

where \( M(u) \) and \( E(u) \) are functions of the magnitude of the velocity \( u \). The relativistic momentum and energy of (2) must reduce to (1) in the nonrelativistic case. Hence

\[ M(0) = m \]  
\[ \frac{\partial E}{\partial u^2}(0) = \frac{m}{2} \]

(3a) \hspace{1cm} (3b)

Our aim is to determine the functional dependence \( M(u) \) and \( E(u) \) on \( u \). We make the assumption that \( M(u) \) and \( E(u) \) are well behaved monotonic functions of \( u \). We then consider the elastic collision of two identical particles and utilize the conservation of momentum and energy which must hold in all equivalent inertial frames, commensurate with the relativity principle. In particular, we consider the collision in two frames \( K \) and \( K' \) related by the ‘modified Lorentz transformation’.

Let the inertial frame \( K' \) have two identical particles having initial velocities \( \vec{u}'_a = \vec{v}, \vec{u}'_b = -\vec{v} \) along the \( Ox \) axis. The particles collide and scatter after which they have final velocities \( \vec{u}'_c = \vec{v}' \) and \( \vec{u}'_d = \vec{v}'' \). The particles and their velocities are depicted in Figure 1.

In \( K' \) the conservation of momentum and energy read:

\[ \vec{p}'_a + \vec{p}'_b = \vec{p}'_c + \vec{p}'_d \]  
\[ E'_a + E'_b = E'_c + E'_d \]

Or with the form (2)

\[ M(v)\vec{v} - M(v')\vec{v}' = M'(v')\vec{v}' + M'(v'')\vec{v}'' \]  
\[ E(v) + E(v') = E'(v') + E'(v'') \]

(4a) \hspace{1cm} (4b)

Because the particles are identical, \( E(v') = E(v'') \) must hold. Since we assumed monotonic behavior for \( E(v) \), then \( v' = v'' \) is true. Then the second equation in (4) requires \( v' = v'' = v \). The first equation requires \( \vec{v}''' = -\vec{v}' \). All four velocities are the same in magnitude with the final velocities opposite in direction, just like the initial velocities. Assume \( \theta' \) represents the scattering angle in \( K' \).

![Figure 1: Initial and final velocity vectors in frame \( K' \) for the collision of two identical particles.](image)

We now consider the same collision in another inertial frame $K'$ moving with a velocity $\vec{v}_2 = -\vec{v}$ in the $OX$ direction with respect to $K'$. From Section 1 where the transform equations for the velocity are given, we observe that particle $a$ moves along the $OX$ axis with velocity given by

$$u_{ax} = \frac{2v}{(c'/c - rv/c)} \quad u_{ay} = 0 \quad u_{az} = 0$$

$$\vec{u}_a = \frac{2\vec{v}}{c' (1 + v^2/c^2) / c} = \frac{2\vec{\beta}}{(1 + \beta^2)}$$

where $\vec{\beta} = \frac{\vec{v}}{c}$ and $\beta = \frac{v}{c}$.

Similarly, because $\vec{u}'_c = \vec{v}' = v' \cos \theta' \vec{i} + v' \sin \theta' \vec{j}$ where $\vec{i}$ and $\vec{j}$ are unit vectors along $O'x'$ and $O'y'$,

$$u_{cx} = \frac{v' (1 + \cos \theta')}{(c' (1 + \beta^2 \cos \theta'))} = \frac{c \beta (1 + \cos \theta')}{(1 + \beta^2 \cos \theta')} \quad u_{cy} = \frac{c \beta \sin \theta'}{\alpha (1 + \beta^2 \cos \theta')}$$

Similarly because $\vec{u}'_d = -\vec{v}' = -(v' \cos \theta' \vec{i} + v' \sin \theta' \vec{j})$

$$u_{dx} = \frac{c \beta (1 - \cos \theta')}{(1 - \beta^2 \cos \theta')} \quad u_{dy} = -\frac{c \beta \sin \theta'}{\alpha (1 - \beta^2 \cos \theta')}$$

with $\alpha = (1 - \beta^2)^{-1/2}$.

The conservation of momentum and energy equations in $K$ are:

$$M(u_a) \vec{u}_a + M(u_b) \vec{u}_b = M(u_c) \vec{u}_c + M(u_d) \vec{u}_d,$$

$$E(u_a) + E(u_b) = E(u_c) + E(u_d).$$

(7a) (7b)

It can be seen from (5) and (6) that while particle $b$ is at rest because

$$u_{bx} = \frac{u_{bx}'}{(c'/c - ru_{bx}' / c)} = \frac{-v + v}{(c'/c + rv/c)} = 0 \quad u_{by} = 0, \quad u_{bz} = 0,$$

the other three velocities are all different in general. Thus the determination of $M(u)$ and $E(u)$ from (7) looks complicated. We can however consider the limiting situation of a glancing collision in which $\theta'$ is very small. Then in the frame $K$, $\vec{u}_d$ will be nonrelativistic and $\vec{u}_c$ will differ only slightly from $\vec{u}_a$. We can therefore make appropriate Taylor series expansions around $\theta' = 0$ and obtain equations involving $M(u)$, $E(u)$ and perhaps their first derivatives.

Explicitly the $y$ component of the momentum conservation equation in (7a) is:

$$0 = M(u_c) \frac{c \beta \sin \theta'}{\alpha (1 + \beta^2 \cos \theta')} - M(u_d) \frac{c \beta \sin \theta'}{\alpha (1 - \beta^2 \cos \theta')},$$

Canceling common terms and rearranging terms we have,

$$M(u_c) = M(u_d) \frac{(1 + \beta^2 \cos \theta')}{(1 - \beta^2 \cos \theta')}.$$ 

This relation is valid for all $\theta'$, and in particular for $\theta' = 0$. Examination of (6) shows that in that limit $u_c = u_a$, $u_d = 0$. Thus we obtain

$$M(u_a) = M(0) \frac{1 + \beta^2}{1 - \beta^2}. \quad (8)$$

From (5) it can easily be observed that

$$\frac{1 + \beta^2}{1 - \beta^2} = \frac{1}{\sqrt{1 - u_a^2/c^2}} = \alpha_a$$

(9)

will hold.
With the value $M(0) = m$ from (3) we thus have

$$M(u_a) = m\alpha_a$$

which implies that the momentum of a particle of mass $m$ and velocity $u$ is

$$\vec{p} = m\vec{u} = \frac{m\vec{u}}{\sqrt{1 - u^2/c^2}} \quad (10)$$

Determining the functional form of $E(u)$ needs examination of the conservation of energy equation for small $\theta'$ rather than at $\theta' = 0$. From (7) we have

$$E(u_a) + E(0) = E(u_c) + E(u_d) \quad (11)$$

where $u_c$ and $u_d$ are functions of $\theta'$. From (6), we find correct to order $\theta'^2$ inclusive,

$$u_{c}^2 = u_{a}^2 - \frac{\eta}{\alpha_a^3} + O(\eta^2)$$

$$u_{d}^2 = \eta + O(\eta^2)$$

where $\alpha_a$ is given by (9) and $\eta$ is defined as $\eta = \frac{\beta^2 - \beta'\theta^2}{1 - \beta^2}$.

In Equation (7b), we equate coefficients of different powers of $\eta$. First order terms yield for $\eta \neq 0$

$$\frac{\partial E(u_c)}{\partial u_c^2} \left(-\frac{1}{\alpha_a^2}\right) + \frac{\partial E(u_d)}{\partial u_d^2} = 0 \quad (12)$$

$$\frac{\partial E(u_c)}{\partial u_c^2} = \frac{\partial E(u_a)}{\partial u_a^2} \quad (13)$$

But $\lim_{\eta \to 0} \frac{\partial E(u_d)}{\partial u_d^2} = \frac{\partial E(u_a)}{\partial u_a^2}(0) = \frac{m}{2}$ as per Equations (3). Utilizing (13) one has

$$\frac{\partial E(u_a)}{\partial u_a^2} = \frac{\partial E(u_c)}{\partial u_c^2} = \alpha_a^3 \frac{\partial E(u_d)}{\partial u_d^2} = \frac{\alpha_a^3 m}{2} = \frac{m}{2 (1 - u_a^2/c^2)^{3/2}}$$

Integration yields:

$$E(u_a) = \frac{mc^2}{(1 - u_a^2/c^2)^{1/2}} - mc^2 + E(0) \quad (14)$$

We have

$$p_0 = \frac{m}{\sqrt{1 - u^2/c^2}}$$

$$c p_0 = \frac{mc^2}{\sqrt{1 - u^2/c^2}}$$

Based on (14)

$$E(u_a) + mc^2 = E(0) + c p_0 \quad (15)$$

The solution to $E(0)$ from this equation is provided in the Appendix and is found to be $E(0) = mc^2$. Hence

$$E(u) = \frac{mc^2}{(1 - u^2/c^2)^{1/2}}.$$  

The corresponding results derived for $K'$ will read

$$E'(0) = mc^2$$

$$E'(u_a') = \frac{mc^2}{\left[1 - (u'^2/c^2)^2\right]^{1/2}} = \frac{mc^2}{(1 - u^2/c^2)^{1/2}}.$$
Appendix: Determination of $E(0)$

We shall utilize the fact that $E(0)$ has to be the same in relativistic and nonrelativistic speeds and compute the quantity $E^2(u) - (pc)^2$, using the expressions $E(u) = E(0) + \frac{1}{2}mu^2$, $E(u) = \frac{mc^2}{\sqrt{1-u^2/c^2}} - mc^2 + E(0)$ and $cp = \frac{mc^2}{\sqrt{1-u^2/c^2}}$. Then

$$E^2(u) - (pc)^2 = E^2(0) + mu^2E(0) + \frac{1}{4} (mu^2)^2 - m^2 \left( \frac{u^2c^2}{1-u^2/c^2} \right)^2$$

$$= m^2c^4 + \frac{2mc^2}{\sqrt{1-u^2/c^2}} [E(0) - mc^2] + [E(0) - mc^2]^2 \quad (A1)$$

must hold for nonrelativistic speeds $u$. We now claim that the solution for above equation is $E(0) = mc^2$. To see this we substitute this value for $E(0)$ in (A1). We get

$$m^2c^4 + mu^2mc^2 + m^2c^4 \left[ \frac{1}{4} \left( \frac{u^2}{c^2} \right)^2 - \left( \frac{u^2/c^2}{1-u^2/c^2} \right) - 1 \right] = 0.$$

In this equation within the brackets, we neglect the term $\frac{1}{4}(\frac{u^2}{c^2})^2$ when compared with 1 and divide the equation by $c^4$ to get;

$$m^2 + m^2u^2/c^2 - m^2 \left( \frac{u^2/c^2}{1-u^2/c^2} \right) - m^2 = 0.$$

Or

$$m^2 \left[ 1 + \frac{u^2}{c^2} - \frac{u^2/c^2}{1-u^2/c^2} \right] - m^2 = 0.$$

This is equivalent to

$$m^2 \left[ \frac{1-u^4/c^4}{1-u^2/c^2} \right] - m^2 = 0.$$

We again neglect the term $(\frac{u^2}{c^2})^2$ when compared with 1.

$$m^2 \left[ \frac{1-u^2/c^2}{1-u^2/c^2} \right] - m^2 = 0$$

which shows that our solution $E(0) = mc^2$ is true.

REFERENCES