Gravitational search algorithm for determination of controller parameters in an AVR system

Serhat DUMAN¹*, Nuran YORUKEREN², Ismail H. ALTAS³
¹Duzce University, Faculty of Technology, Electrical & Electronics Engineering Department, Duzce, Turkey.
²Kocaeli University, Faculty of Engineering, Electrical Engineering Department, Kocaeli, Turkey.
³Karadeniz Technical University, Faculty of Engineering, Electrical & Electronics Engineering Department, Trabzon, Turkey.
*serhatduman@duzce.edu.tr

Abstract – This paper presents optimal tuning of the controller parameters of a proportional-integral-derivate (PID) controller for an Automatic Voltage Regulator (AVR) system using heuristic Gravitational Search Algorithm (GSA) based on mass interactions and the Newton’s Law of Gravity. Determination of the optimal controller parameters is considered as an optimization problem. In this optimization problem, different performance indexes and a performance criterion in the time domain have been used as objective function to test performance and the effectiveness of the GSA. In the determining process of the parameters, the designed PID controller with the proposed approach is simulated under different conditions and performance of the controller is compared to those reported in literature. From numerical simulation results, it is clearly shown that the GSA approach is successfully applied to reveal the performance and the feasibility of the proposed controller in the AVR system.

Keywords Gravitational Search Algorithm, Optimization, Automatic Voltage Regulator

1. Introduction
The proportional-integral-derivate (PID) controller is the most widely used control law in engineering field. In process control, more than 90% of the control loops are the PID controller. It is quite obvious that PID controller is widely used due to their simple structure and high performance in a wide range of operating conditions [1,2]. The PID controller exhibits relatively weak dynamic performance as evidenced by large overshoot and transient frequency oscillations. To design a PID controller is to specify parameters as $K_p$, $K_i$ and $K_d$
Regrettably, it has been fairly difficult to tune the gains of PID controller properly in industrial operations. In recent years, modern heuristic optimization techniques are proposed to tune the PID controller parameters instead of traditional methods that are inadequate because of possible changes in operating conditions. Conventional methods can be listed as Ziegler-Nichols, gain-phase margin method and Cohen-Coon [4]. Heuristic methods are such as genetic algorithms [5-9], evolutionary algorithms [10-11], modified ant colony optimization algorithm based on differential evolution [12], incremental learning algorithm [13], particle swarm algorithm (PSO) [14-16] and ant colony algorithm (ACO) [17-19].

**Automatic Voltage Regulator (AVR)** uses the exciter voltage of a generator which is responsible for keeping the terminal voltage magnitude of a synchronous generator constant under normal operating conditions at different load levels [2]. Kim and Park have determined the optimal PID controller parameters using meta-heuristic methods. Results obtained from simulation study show that hybrid system composed from EU-GA-PSO is more satisfactory than Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) [20]. Zhu et al proposed a chaotic ant swarm algorithm for design of the controller in the AVR system. Designing of the PID controller with chaotic ant swarm algorithm has been effective to improve stability of system according to results obtained from many simulation examples [21]. Mukherjee and Ghoshal explored the specifications of the optimal PID controller parameters using craziness based and velocity relaxed swarm optimization algorithm [22]. Kim designed a PID controller based hybrid GA-BF for automatic voltage regulator. In optimization process, suggested approach has been more effective from GA, PSO and GA-PSO to tune of PID controller parameters [23]. Gozde and Taplamacioglu have investigated performance analysis of Artificial Bee Colony (ABC) for an AVR system. They say ABC is applied to different control applications [24]. Mukherjee and Ghoshal used heuristic methods to tune the PID controller parameter for an AVR. They claim that PID controller based on CRPSO-SLF provides better performance for step response of terminal voltage with less computational effort compared to the other heuristic algorithms [25]. Zonkoly used PSO algorithm to tune the parameters of a coordinated power system stabilizer and the AVR in a multi-machine power system. Performance of the proposed method is compared to some heuristic methods and mathematical optimization algorithms such as genetic algorithm, quadratic programming methods and linear programming, respectively and proved to be efficient in determining the optimal values of the control parameters [26]. Design of an AVR system using the PSO heuristic optimization method was presented Zamani et al. The proposed controller with the PSO algorithm has better performance characteristic and stability than the traditional controller under various scenarios [27]. Genetic algorithm and bacterial
foraging based on a novel hybrid approach was presented by Kim et al. This hybrid approach (GA-BF) is tested using various test functions and is used to tune the parameters of a PID controller of an AVR system [28].

Among the available meta-heuristics algorithms, the GSA, one of the lately improved heuristic algorithms, based on the Newton’s law of gravity and mass interaction is proposed by Rashedi et al [29]. Masses are regarded as individuals of the population in this approach. GSA has a simple structure and an effective calculation ability. Exploration and exploitation abilities can be improved for its flexible and well-balanced structure [30]. The gravitational constant is decreased with time to arrange the accuracy of the search, which is defined as the most significant feature of the GSA. Thus, solution process of the GSA is accelerated [31-32]. Also, algorithm needs less memory [32]. Nowadays, many researchers have used this algorithm for solving various problems in the literature [33-40].

In this study, performance analysis of the proposed method is tested to tune the gains of the PID controller in a practical high-order AVR. Integral time squared error (ITSE), integral time absolute error (ITAE), integral squared error (ISE), integral absolute error (IAE) and a performance creation in the time domain are used as objective function to test the performance of the proposed approach. The designed PID controller with the proposed heuristic approach is simulated under various operating conditions and performance of the controller is compared with those reported in [24].

2. Model of an AVR System

The PID controller is widely used to improve the dynamic response of the system and to decrease the steady-state error. The proportional part of the controller is used to reduce error under disturbances conditions. An enhancement of the transient response and stability of the system is achieved by the derivative part of the controller which adds a finite zero to the open-loop plant transfer function of the system. Integral part of controller which adds a pole to the origin and increases the system type by one is used to eliminate steady-state error [2,4,24]. Equation (1) shows transfer function of the PID controller.

\[ G(s) = K_p + \frac{K_i}{s} + K_d s \]  

(1)

The AVR system plays an important role to keep the terminal voltage of generator at a specified level. Normal and fault conditions of operation are taken into account to design the AVR. Therefore, security of the power system is seriously affected by stability of the AVR system. The real model of this system is depicted in Figure (1) [24]. The AVR system
involves of generators, amplifiers, exciters and sensors, which they are defined the four main components in an AVR system. The reasonable transfer function of these components using the PID controller is shown in Figure (2) [1,2,4,21].

![Diagram of AVR system components](image)

**Figure 1.** The model of the AVR system

![Block diagram with transfer function model of the system](image)

**Figure 2.** Block diagram with transfer function model of the system

The components of the AVR system and the boundary values of the system are described in Table 1. The transfer function of the system with the controller is described in (2).

\[
\frac{\Delta V_r(s)}{\Delta V_{ref}(s)} = \frac{(s^2 K_d + sK_p + K_i)(K_a K_e K_g)(1 + sT_r)}{s(1 + sT_a)(1 + sT_e)(1 + sT_g)(1 + sT_r) + (K_a K_e K_g K_i)(s^2 K_d + sK_p + K_i)}
\]  

(2)
Table 1. Parameter limits of the AVR system and the PID controller

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter Limits</th>
<th>Used parameter values in AVR system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controller</td>
<td>0.2\leq K_p, K_i, K_d \leq 2.0</td>
<td>Optimal values (K_p, K_i, K_d)</td>
</tr>
<tr>
<td>Amplifier</td>
<td>10 \leq K_a \leq 40 \quad 0.02 \leq T_a \leq 0.1</td>
<td>K_a=10, T_a=0.1</td>
</tr>
<tr>
<td>Exciter</td>
<td>1 \leq K_e \leq 10 \quad 0.4 \leq T_e \leq 1.0</td>
<td>K_e=1, T_e=0.4</td>
</tr>
<tr>
<td>Generator</td>
<td>K_g (0.7-1.0) \quad 1.0 \leq T_g \leq 2.0</td>
<td>K_g=1, T_g=1</td>
</tr>
<tr>
<td>Sensor</td>
<td>0.001 \leq T_s \leq 0.06</td>
<td>K_s=1, T_s=0.01</td>
</tr>
</tbody>
</table>

3. Gravitational Search Algorithm

The heuristic optimization method was firstly improved by Rashedi et al., which is motivated by the Newton’s Laws of Gravity and Motion [29]. The algorithm has many advantages that are reported in [29] and authors have compared the GSA with other stochastic methods using 23 benchmark test functions and they have inferred that GSA is stronger than compared with those methods. In this approach, entire agents are used as objects and their performance that will be computed by using a fitness function are denoted by their masses. The gravitational force is attracted every object to other objects. Motion of entire agents globally towards the agents with heavier masses is provided by this force. The heavy masses are described as good solutions of the optimization problem [29]. The proposed algorithm can be depicted as following:

At the beginning of the algorithm variables are described with $M$ masses.

$$X_i = (x_i^1, x_i^2, ..., x_i^M) \quad \text{for, } i=1,2,...,M$$  \hspace{1cm} (3)

where $x_i^d$ is position of the $i^{th}$ mass in the $d^{th}$ dimension and $M$ is dimension of the search space. best and worst fitness values according to minimization or maximization problem are described:

Minimization problem is defined as follows:

$$\text{best}(k) = \min_{j=1,...,M} fit_j(k)$$ \hspace{1cm} (4)

$$\text{worst}(k) = \max_{j=1,...,M} fit_j(k)$$ \hspace{1cm} (5)
And maximization problem is:

\[
\text{best}(k) = \max_{j \in [1, M]} \text{fit}_j(k) \tag{6}
\]

\[
\text{worst}(k) = \min_{j \in [1, M]} \text{fit}_j(k) \tag{7}
\]

where \( \text{fit}_j(k) \) is defined as the fitness value of the \( j^{th} \) agent at time \( k \). \( \text{best}(k) \) fitness and \( \text{worst}(k) \) fitness values are indicate the powerful and the powerless agent for maximization or minimization problem in the search space. The gravitational constant is computed in (8)-(9).

\[
G(k) = G(G_0, k) \tag{8}
\]

\[
G(k) = G_0 e^{-\frac{k}{K}} \tag{9}
\]

Gravitational constant \( (G_0) \) will be decreased with time to adjusts the accuracy of the search. The initial value of the \( G_0 \) and \( \alpha \) are specified by user. \( k \) and \( K \) are the current iteration, and the total number iterations, respectively. Inertial masses are defined for each agent at iteration.

\[
M_{pi} = M_{ai} = M_{ii} = M_i \tag{10}
\]

\[
m_i(k) = \frac{\text{fit}_i(k) - \text{worst}(k)}{\text{best}(k) - \text{worst}(k)} \tag{11}
\]

\[
M_i(k) = \frac{m_i(k)}{\sum_{j=1}^{M} m_j(k)} \tag{12}
\]

where \( M_{ai}, M_{pi}, M_{ii} \) and \( M_i(k) \) are the active mass, the passive mass, the inertia mass of the \( i^{th} \) agents and the mass of the \( i^{th} \) agent at iteration \( k \), respectively. The sum of force acting on the \( i^{th} \) agent \( (F_i^{d}(k)) \) is computed as in (13).

\[
F_i^{d}(k) = \sum_{j \in \text{best(i)}} \text{rand}_j F_j^{d}(k) \tag{13}
\]

\( \text{rand}_j \) is a randomly defined number in interval \([0,1]\). The force acting on the \( i^{th} \) mass \( (M_i(k)) \) from the \( j^{th} \) mass \( (M_j(k)) \) at the current iteration \( k \) is defined according to the gravitational
theory. Mathematical equation of this theory is offered in (14). Figure (3) shows the sum of the forces acting on an object.

\[
F^d_i(k) = G(k) \frac{M_j(k) \times M_j(k)}{R_{ij}(k)} \left( x^d_i(k) - x^d_j(k) \right)
\] (14)

\(R_{ij}(k)\) is the Euclidean distance between agents \(i^{th}\) and \(j^{th}\), Euclidean distance is defined as 
\(R_{ij}(k) = \left\| X_i(k) - X_j(k) \right\|_2\) and \(\epsilon\) is the small constant.

![Figure 3. Total the forces acting on an object](image)

In an attempt to find the acceleration of the \(i^{th}\) agent, at \(t\) time in the \(d^{th}\) dimension based on law of motion is used directly to calculate. \(a^d_i(k)\) is expressed as in (15)-(16).

\[
a^d_i(k) = \frac{F^d_i(k)}{M_i(k)}
\] (15)

\[
a^d_i(k) = \frac{F^d_i(k)}{M_i(k)} = \sum_{j \neq \text{closest}\ i} \text{rand} \times G(k) \frac{M_j(k)}{R_{ij}(k)} \left( x^d_j(k) - x^d_i(k) \right)
\] (16)

The velocity of an agent is identified as a function of its acceleration value of the agent added to its velocity. The new velocity of an agent is obtained as in (17).

\[
v^d_i(k+1) = \text{rand} \times v^d_i(k) + a^d_i(k)
\] (17)
Here, $rand_i$ is a number that is randomly distributed in interval $[0,1]$. The new position of the $i^{th}$ agents in $d^{th}$ dimension are expressed as in (18).

$$x_i^d (k+1) = v_i^d (k+1) + x_i^d (k)$$  \hspace{1cm} (18)$$

GSA algorithm flowchart is illustrated in Figure (4).

**Figure 4.** The flowchart of the GSA [29].

4. Simulation Results

In this study, GSA heuristic approach is used to tune optimal parameters of the PID controller of an AVR system. In order to examine into the performance of the offered heuristic approach, it is compared with [24] and ABC algorithm under various operating scenarios. In this study, ITSE and a performance criterion in the time domain are used to tune optimal values of the controller parameters as objective function, which is represented in Eq. 19 and 20, respectively. The performance criterion in the time domain contain steady state error $E_{SSS}$, rise time $t_r$, the settling time $t_s$, and overshoot $M_p$. $\beta$ is weighting factor, which selected from 0.5 to 1.5 in steps of 0.5 in this study. Moreover, the time constants of the AVR system are
changed together in the range of $+25\% - +100\%$ in order to analyze the robustness of the proposed stochastic optimization algorithm.

In order to demonstrate the efficiency and robustness of the proposed algorithm, it is applied to different objective functions such as IAE, ISE, ITAE and ITSE. Objective functions are shown in (19) and (21-23). Block diagram of an AVR system with the optimized PID controller using the proposed heuristic approach is shown in Figure (5).

\[ J = ITSE = \int_0^t e^2(t)dt \]  \hspace{1cm} (19)

\[ \text{Min } J(K_p, K_i, K_d) = (1 - e^{-\beta})(M_s + E_s) + e^{-\beta}(t_s - t_r) \]  \hspace{1cm} (20)

\[ J = IAE = \int_0^t |e(t)|dt = \int_0^t |r(t) - y(t)|dt \]  \hspace{1cm} (21)

\[ J = ISE = \int_0^t e^2(t)dt \]  \hspace{1cm} (22)

\[ J = ITAE = \int_0^t |t|e(t)|dt \]  \hspace{1cm} (23)

![Figure 5. An AVR system with proposed controller](image)

The size of the population and number of iteration are chosen the same for all heuristic approaches. Also, $G_0$ and $\alpha$ parameters of the GSA approach are taken as 200 and 20, respectively. The obtained optimal values of the PID controller parameters, response of the AVR system at the end of the simulation process are represented in Table 2. Transfer functions of the AVR system for ABC, PSO and Differential Evolutionary (DE) are shown in...
(24), (25) and (26), respectively. Transfer function of the system adjusted by the GSA approach is shown in (27).

\[
\frac{\Delta \bar{V}_p(s)}{\Delta \bar{V}_{ref}(s)} = \frac{0.03654s^3 + 3.819s^2 + 16.56s + 4.083}{0.0004s^5 + 0.0454s^4 + 0.555s^3 + 5.164s^2 + 17.52s + 4.083} \tag{24}
\]

\[
\frac{\Delta \bar{V}_i(s)}{\Delta \bar{V}_{ref}(s)} = \frac{0.03184s^3 + 3.362s^2 + 17.81s + 3.827}{0.0004s^5 + 0.0454s^4 + 0.555s^3 + 4.694s^2 + 18.77s + 3.827} \tag{25}
\]

\[
\frac{\Delta \bar{V}_d(s)}{\Delta \bar{V}_{ref}(s)} = \frac{0.03427s^3 + 3.622s^2 + 19.54s + 4.43}{0.0004s^5 + 0.0454s^4 + 0.555s^3 + 4.937s^2 + 20.05s + 4.43} \tag{26}
\]

\[
\frac{\Delta \bar{V}_o(s)}{\Delta \bar{V}_{ref}(s)} = \frac{0.07363s^3 + 7.507s^2 + 14.5s + 12.21}{0.0004s^5 + 0.0454s^4 + 0.555s^3 + 8.873s^2 + 15.38s + 12.21} \tag{27}
\]

**Table 2.** Optimized PID parameters and transient response parameters

<table>
<thead>
<tr>
<th>$K_g$</th>
<th>$T_g$</th>
<th>Type of Controller</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>Max. Overshoots</th>
<th>Settling times (5% band)</th>
<th>Rise Times</th>
<th>Peak Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>ABC-PID [24]</td>
<td>1.6524</td>
<td>0.4083</td>
<td>0.3654</td>
<td>1.250</td>
<td>0.920</td>
<td>0.156</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PSO-PID [24]</td>
<td>1.7774</td>
<td>0.3827</td>
<td>0.3184</td>
<td>1.300</td>
<td>1.000</td>
<td>0.161</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DE-PID [24]</td>
<td>1.9499</td>
<td>0.4430</td>
<td>0.3427</td>
<td>1.330</td>
<td>0.952</td>
<td>0.152</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GSA-PID</td>
<td>1.4379</td>
<td>1.2208</td>
<td>0.7363</td>
<td>1.240</td>
<td>0.597</td>
<td>0.107</td>
<td>0.24</td>
</tr>
</tbody>
</table>

It is clear from Table 2 that GSA algorithm has better performance for percent overshoots as 0.8065 % than ABC, 4.8387% PSO and 7.2580% than DE algorithms, respectively. When the GSA algorithm is examined in terms of peak time, it provides 50% better results than ABC and DE algorithms, 58.3333% compared to PSO algorithm. For settling time, the proposed algorithm has better results as 54.1038% than ABC algorithm, 59.4639% than DE algorithm and 67.5041% than PSO algorithm. It appears that the rise time of the GSA algorithm has the best result as 50.4672%, 42.056%, 45.7943% than other PSO, DE and ABC heuristic methods, respectively. The results obtained by changing the voltage curve at the end of the simulation of the GSA are presented comparatively for ABC, PSO and DE [24] in Figure (6) and Figure (7), which shows settling times for each, which is within 5% bandwidth.
**Figure 6.** Voltage changing curves of the GSA, ABC, PSO and DE algorithms

**Figure 7.** Zoom of voltage changing curves for settling times of heuristic algorithms
In this section of the study, performance criterion in the time domain is used to tune parameters of the controller as objective function for the results in Table 3. The obtained optimal gain values of the PID controller by the GSA approach for different values of weighting factor are given in Table 3.

In Table 3, it is seen that the performance analysis results obtained from the proposed GSA-PID approach are compared with the results obtained from the ABC-PID approach. The bound values of the PID controller parameters are chosen in between [0,1]. The results of the comparison demonstrate that the designed PID controller by the proposed approach has less settling time, rise time and overshoot than ABC-PID controller. Figure (8) shows the behaviors of the AVR system with tuned gains by GSA and ABC for different weighting factors. The transient response of the system has been effectively improved by the proposed approach and comparison of the responses of the heuristic approaches are shown in Figure (8).

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Type of Controller</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>Max. Overshoots (%)</th>
<th>Settling times (5% band)</th>
<th>Rise Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>GSA-PID</td>
<td>0.6976</td>
<td>0.6027</td>
<td>0.3376</td>
<td>1.46</td>
<td>0.838</td>
<td>0.214</td>
</tr>
<tr>
<td></td>
<td>ABC-PID</td>
<td>0.7320</td>
<td>0.2827</td>
<td>0.2191</td>
<td>2.55</td>
<td>1.31</td>
<td>0.278</td>
</tr>
<tr>
<td>1</td>
<td>GSA-PID</td>
<td>0.6068</td>
<td>0.4465</td>
<td>0.1897</td>
<td>2.04</td>
<td>0.447</td>
<td>0.325</td>
</tr>
<tr>
<td></td>
<td>ABC-PID</td>
<td>0.6806</td>
<td>0.2183</td>
<td>0.1381</td>
<td>6.63</td>
<td>2</td>
<td>0.34</td>
</tr>
<tr>
<td>1.5</td>
<td>GSA-PID</td>
<td>0.6587</td>
<td>0.7626</td>
<td>0.4088</td>
<td>3.16</td>
<td>0.814</td>
<td>0.186</td>
</tr>
<tr>
<td></td>
<td>ABC-PID</td>
<td>0.6705</td>
<td>0.9679</td>
<td>0.3479</td>
<td>5.15</td>
<td>1.76</td>
<td>0.209</td>
</tr>
</tbody>
</table>

Table 3. Comparison of the results from the GSA and ABC approaches
(a)

(b)
Figure 8. Voltage changing curves of the GSA and ABC: (a) for $\beta = 0.5$ (b) for $\beta = 1$ (c) for $\beta = 1.5$

When the gains of the AVR system is fixed, the entire time constants of the AVR system is changed in the range of +25% - +100% to evaluate the performance and robustness of the heuristic optimization algorithm. The obtained results of the proposed approach are shown in Figure (9) and Table 4.

**Table 4. Results of the GSA approach for entire time constants**

<table>
<thead>
<tr>
<th>Time Constants</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>Max. Overshoots (%)</th>
<th>Settling times (5 % band)</th>
<th>Rise Times</th>
<th>Peak Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>+25%</td>
<td>1.3331</td>
<td>0.9864</td>
<td>0.9833</td>
<td>23.4</td>
<td>0.731</td>
<td>0.1285</td>
<td>0.295</td>
</tr>
<tr>
<td>+50%</td>
<td>1.1866</td>
<td>1.0379</td>
<td>1.1466</td>
<td>21.5</td>
<td>0.876</td>
<td>0.1590</td>
<td>0.362</td>
</tr>
<tr>
<td>+75%</td>
<td>1.1885</td>
<td>0.9422</td>
<td>1.2683</td>
<td>20.7</td>
<td>1.032</td>
<td>0.1923</td>
<td>0.421</td>
</tr>
<tr>
<td>+100%</td>
<td>1.0157</td>
<td>1.0613</td>
<td>1.7766</td>
<td>23.4</td>
<td>1.09</td>
<td>0.1922</td>
<td>0.438</td>
</tr>
</tbody>
</table>
Figure 9. Voltage change curves ranging from +25% to +100% for entire time constants

Also, to demonstrate the efficiency of the GSA approach, different performance indexes such as IAE, ISE, ITAE and ITSE are used as objective function. These performance indexes are utilized under various operating conditions. The obtained simulation results of the proposed GSA heuristic algorithm are shown in Table 5 and comparison of the performance indexes is shown in Figures (10)-(13). From Table 5, it appears that the ITAE performance index has better performance for percent overshoots and when settling times are investigated, the best results belong to ITSE and IAE performance indexes under 0.7-0.8, 0.9-1.0 operating conditions, respectively. ISE performance index has better performance for rise times and peak times than other performance indexes under different operating conditions.
Table 5. Results of the GSA approach for different performance indexes

<table>
<thead>
<tr>
<th>$K_g$</th>
<th>Objective Functions</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>Max. Overshoots (%)</th>
<th>Settling times (5 % band)</th>
<th>Rise Times</th>
<th>Peak Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>IAE</td>
<td>2.0000</td>
<td>1.4668</td>
<td>0.8639</td>
<td>20.6</td>
<td>0.650</td>
<td>0.1219</td>
<td>0.272</td>
</tr>
<tr>
<td></td>
<td>ISE</td>
<td>1.6889</td>
<td>2.0000</td>
<td>1.9785</td>
<td>36.4</td>
<td>0.769</td>
<td>0.0702</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td>ITAE</td>
<td>1.9656</td>
<td>1.3511</td>
<td>0.6335</td>
<td>19.3</td>
<td>0.699</td>
<td>0.1475</td>
<td>0.329</td>
</tr>
<tr>
<td></td>
<td>ITSE</td>
<td>1.9398</td>
<td>1.7845</td>
<td>1.0851</td>
<td>23.1</td>
<td>0.591</td>
<td>0.1050</td>
<td>0.236</td>
</tr>
<tr>
<td>0.8</td>
<td>IAE</td>
<td>2.0000</td>
<td>1.4206</td>
<td>0.8825</td>
<td>24.9</td>
<td>0.608</td>
<td>0.1082</td>
<td>0.245</td>
</tr>
<tr>
<td></td>
<td>ISE</td>
<td>1.4473</td>
<td>2.0000</td>
<td>1.7424</td>
<td>36.6</td>
<td>0.761</td>
<td>0.0700</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td>ITAE</td>
<td>1.8063</td>
<td>1.2567</td>
<td>0.6446</td>
<td>20.5</td>
<td>0.682</td>
<td>0.1338</td>
<td>0.302</td>
</tr>
<tr>
<td></td>
<td>ITSE</td>
<td>1.8031</td>
<td>1.8250</td>
<td>1.0181</td>
<td>25.4</td>
<td>0.567</td>
<td>0.0995</td>
<td>0.230</td>
</tr>
<tr>
<td>0.9</td>
<td>IAE</td>
<td>2.0000</td>
<td>1.5909</td>
<td>1.1247</td>
<td>32.5</td>
<td>0.512</td>
<td>0.0849</td>
<td>0.208</td>
</tr>
<tr>
<td></td>
<td>ISE</td>
<td>0.8296</td>
<td>2.0000</td>
<td>1.6192</td>
<td>35.8</td>
<td>0.757</td>
<td>0.0685</td>
<td>0.160</td>
</tr>
<tr>
<td></td>
<td>ITAE</td>
<td>1.9334</td>
<td>1.3627</td>
<td>0.6536</td>
<td>25.9</td>
<td>0.649</td>
<td>0.1195</td>
<td>0.279</td>
</tr>
<tr>
<td></td>
<td>ITSE</td>
<td>1.4250</td>
<td>1.4025</td>
<td>0.8732</td>
<td>22.9</td>
<td>0.585</td>
<td>0.1032</td>
<td>0.235</td>
</tr>
<tr>
<td>1.0</td>
<td>IAE</td>
<td>1.9105</td>
<td>1.3435</td>
<td>0.8359</td>
<td>30.4</td>
<td>0.559</td>
<td>0.0952</td>
<td>0.232</td>
</tr>
<tr>
<td></td>
<td>ISE</td>
<td>1.1383</td>
<td>2.0000</td>
<td>1.4091</td>
<td>36.8</td>
<td>0.747</td>
<td>0.0694</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td>ITAE</td>
<td>1.3029</td>
<td>0.9045</td>
<td>0.4269</td>
<td>17.8</td>
<td>0.687</td>
<td>0.1530</td>
<td>0.336</td>
</tr>
<tr>
<td></td>
<td>ITSE</td>
<td>1.4379</td>
<td>1.2208</td>
<td>0.7363</td>
<td>23.5</td>
<td>0.597</td>
<td>0.1070</td>
<td>0.239</td>
</tr>
</tbody>
</table>

Figure 10. Comparison of performance indexes for $K_g=0.7$
Figure 11. Comparison of performance indexes for $K_g=0.8$

Figure 12. Comparison of performance indexes for $K_g=0.9$
Figure 13. Comparison of performance indexes for $K_g=1.0$

5. Conclusion

This paper focuses on the GSA based on the Newton’s law of gravity and mass interactions that is one of the recently improved heuristic algorithms for possible use of it to tune the PID controller gains. The proposed method was applied to tune optimal gains of the controller in an AVR system. The robustness and performance analysis of this method is tested under various operating conditions. The performance of the stochastic optimization method is compared with [24], and ABC. The robustness of the proposed approach is proved by change of the time constants in the range of +25% - +100% when various performance indexes are used as objective function. The obtained simulation results showed that when the system parameters are changed, the proposed GSA approach can obtain higher quality solution and can be successfully applied to optimize parameters of the PID controller of an AVR system. The obtained dynamic performance of the AVR system from the proposed approach is better compared to the other heuristic approaches. Additionally, the proposed heuristic method according to results obtained from the GSA-PID approach is an influential search method for the optimal gain values of the PID controller. The optimized gains of the PID controller with the GSA approach can be used to improve the performance of the system and to reinforce system stability.
References


[38] A. Chatterjee, G. K. Mahanti, P. R. S. Mahapatra, Generation of phase-only pencil-beam pair from concentric ring array antenna using gravitational search algorithm,


Figure 1. The model of the AVR system

Figure 2. Block diagram with transfer function model of the system

Figure 3. Total the forces acting on an object
Generate initial population
Evaluate fitness of all agents
Compute the $G(t)$, best$(t)$ and worst$(t)$ of the population
Calculate the $M_i(t)$ and $a_i(t)$ for each agent
Update the $v_i(t)$ and $x_i(t)$
Meeting end of criterion ?
Yes
Return best solution

Figure 4. The flowchart of the GSA [29].

$$K_p + K_i + K_d \frac{s}{s+1}$$

Controller

Objective Function

GSA

Amplifier

Exciter

Generator

$V_{ref}(s)$

$V_s(s)$

$V_t(s)$

$\Delta V_r(s)$

$\Delta V_s(s)$

$\Delta V_t(s)$

Sensor

Figure 5. An AVR system with proposed controller
Figure 6. Voltage changing curves of the GSA, ABC, PSO and DE algorithms

Figure 7. Zoom of voltage changing curves for settling times of heuristic algorithms
Figure 8. Voltage changing curves of the GSA and ABC: (a) for $\beta = 0.5$ (b) for $\beta = 1$
(c) for $\beta = 1.5$

Figure 9. Voltage change curves ranging from $+25\%$ to $+100\%$ for entire time constants
Figure 10. Comparison of performance indexes for $K_g=0.7$

Figure 11. Comparison of performance indexes for $K_g=0.8$
Figure 12. Comparison of performance indexes for $K_g=0.9$

Figure 13. Comparison of performance indexes for $K_g=1.0$
Table 1. Parameter limits of the AVR system and the PID controller

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter Limits</th>
<th>Used parameter values in AVR system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controller</td>
<td>0.2 ≤ K_p, K_i, K_d ≤ 2.0</td>
<td>Optimal values (K_p, K_i, K_d)</td>
</tr>
<tr>
<td>Amplifier</td>
<td>10 ≤ K_a ≤ 40, 0.02 ≤ T_a ≤ 0.1</td>
<td>K_a=10, T_a=0.1</td>
</tr>
<tr>
<td>Exciter</td>
<td>1 ≤ K_e ≤ 10, 0.4 ≤ T_e ≤ 1.0</td>
<td>K_e=1, T_e=0.4</td>
</tr>
<tr>
<td>Generator</td>
<td>K_g (0.7-1.0), 1.0 ≤ T_g ≤ 2.0</td>
<td>K_g=1, T_g=1</td>
</tr>
<tr>
<td>Sensor</td>
<td>0.001 ≤ T_s ≤ 0.06</td>
<td>K_s=1, T_s=0.01</td>
</tr>
</tbody>
</table>

Table 2. Optimized PID parameters and transient response parameters

<table>
<thead>
<tr>
<th>K_g</th>
<th>T_g</th>
<th>Type of Controller</th>
<th>K_p</th>
<th>K_i</th>
<th>K_d</th>
<th>Max. Overshoots</th>
<th>Settling times (5% band)</th>
<th>Rise Times</th>
<th>Peak Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>ABC-PID [24]</td>
<td>1.6524</td>
<td>0.4083</td>
<td>0.3654</td>
<td>1.250</td>
<td>0.920</td>
<td>0.156</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PSO-PID [24]</td>
<td>1.7774</td>
<td>0.3827</td>
<td>0.3184</td>
<td>1.300</td>
<td>1.000</td>
<td>0.161</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DE-PID [24]</td>
<td>1.9499</td>
<td>0.4430</td>
<td>0.3427</td>
<td>1.330</td>
<td>0.952</td>
<td>0.152</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GSA-PID</td>
<td>1.4379</td>
<td>1.2208</td>
<td>0.7363</td>
<td>1.240</td>
<td>0.597</td>
<td>0.107</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 3. Comparison of the results from the GSA and ABC approaches

<table>
<thead>
<tr>
<th>β</th>
<th>Type of Controller</th>
<th>K_p</th>
<th>K_i</th>
<th>K_d</th>
<th>Max. Overshoots (%)</th>
<th>Settling times (5% band)</th>
<th>Rise Times</th>
<th>Peak Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>GSA-PID</td>
<td>0.6976</td>
<td>0.6027</td>
<td>0.3376</td>
<td>1.46</td>
<td>0.838</td>
<td>0.214</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ABC-PID</td>
<td>0.7320</td>
<td>0.2827</td>
<td>0.2191</td>
<td>2.55</td>
<td>1.31</td>
<td>0.278</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>GSA-PID</td>
<td>0.6068</td>
<td>0.4465</td>
<td>0.1897</td>
<td>2.04</td>
<td>0.447</td>
<td>0.325</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ABC-PID</td>
<td>0.6806</td>
<td>0.2183</td>
<td>0.1381</td>
<td>6.63</td>
<td>2</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>GSA-PID</td>
<td>0.6587</td>
<td>0.7626</td>
<td>0.4088</td>
<td>3.16</td>
<td>0.814</td>
<td>0.186</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ABC-PID</td>
<td>0.6705</td>
<td>0.9679</td>
<td>0.3479</td>
<td>5.15</td>
<td>1.76</td>
<td>0.209</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Results of the GSA approach for entire time constants

<table>
<thead>
<tr>
<th>Time Constants</th>
<th>K_p</th>
<th>K_i</th>
<th>K_d</th>
<th>Max. Overshoots (%)</th>
<th>Settling times (5% band)</th>
<th>Rise Times</th>
<th>Peak Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>+25%</td>
<td>1.3331</td>
<td>0.9864</td>
<td>0.9833</td>
<td>23.4</td>
<td>0.731</td>
<td>0.1285</td>
<td>0.295</td>
</tr>
<tr>
<td>+50%</td>
<td>1.1866</td>
<td>1.0379</td>
<td>1.1466</td>
<td>21.5</td>
<td>0.876</td>
<td>0.1590</td>
<td>0.362</td>
</tr>
<tr>
<td>+75%</td>
<td>1.1885</td>
<td>0.9422</td>
<td>1.2683</td>
<td>20.7</td>
<td>1.032</td>
<td>0.1923</td>
<td>0.421</td>
</tr>
<tr>
<td>+100%</td>
<td>1.0157</td>
<td>1.0613</td>
<td>1.7766</td>
<td>23.4</td>
<td>1.09</td>
<td>0.1922</td>
<td>0.438</td>
</tr>
</tbody>
</table>
Table 5. Results of the GSA approach for different performance indexes

<table>
<thead>
<tr>
<th>$K_g$</th>
<th>Objective Functions</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>Max. Overshoots (%)</th>
<th>Settling times (5 % band)</th>
<th>Rise Times</th>
<th>Peak Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>IAE</td>
<td>2.0000</td>
<td>1.4668</td>
<td>0.8639</td>
<td>20.6</td>
<td>0.650</td>
<td>0.1219</td>
<td>0.272</td>
</tr>
<tr>
<td></td>
<td>ISE</td>
<td>1.6889</td>
<td>2.0000</td>
<td>1.9785</td>
<td>36.4</td>
<td>0.769</td>
<td><strong>0.0702</strong></td>
<td><strong>0.175</strong></td>
</tr>
<tr>
<td></td>
<td>ITAE</td>
<td>1.9656</td>
<td>1.3511</td>
<td>0.6335</td>
<td><strong>19.3</strong></td>
<td>0.699</td>
<td>0.1475</td>
<td>0.329</td>
</tr>
<tr>
<td></td>
<td>ITSE</td>
<td>1.9398</td>
<td>1.7845</td>
<td>1.0851</td>
<td>23.1</td>
<td><strong>0.591</strong></td>
<td>0.1050</td>
<td>0.236</td>
</tr>
<tr>
<td>0.8</td>
<td>IAE</td>
<td>2.0000</td>
<td>1.4206</td>
<td>0.8825</td>
<td>24.9</td>
<td>0.608</td>
<td>0.1082</td>
<td>0.245</td>
</tr>
<tr>
<td></td>
<td>ISE</td>
<td>1.4473</td>
<td>2.0000</td>
<td>1.7424</td>
<td>36.6</td>
<td>0.761</td>
<td><strong>0.0700</strong></td>
<td><strong>0.175</strong></td>
</tr>
<tr>
<td></td>
<td>ITAE</td>
<td>1.8063</td>
<td>1.2567</td>
<td>0.6446</td>
<td><strong>20.5</strong></td>
<td>0.682</td>
<td>0.1338</td>
<td>0.302</td>
</tr>
<tr>
<td></td>
<td>ITSE</td>
<td>1.8031</td>
<td>1.8250</td>
<td>1.0181</td>
<td>25.4</td>
<td><strong>0.567</strong></td>
<td>0.0995</td>
<td>0.230</td>
</tr>
<tr>
<td>0.9</td>
<td>IAE</td>
<td>2.0000</td>
<td>1.5909</td>
<td>1.1247</td>
<td>32.5</td>
<td><strong>0.512</strong></td>
<td>0.0849</td>
<td>0.208</td>
</tr>
<tr>
<td></td>
<td>ISE</td>
<td>0.8296</td>
<td>2.0000</td>
<td>1.6192</td>
<td>35.8</td>
<td>0.757</td>
<td><strong>0.0685</strong></td>
<td><strong>0.160</strong></td>
</tr>
<tr>
<td></td>
<td>ITAE</td>
<td>1.9334</td>
<td>1.3627</td>
<td>0.6536</td>
<td>25.9</td>
<td>0.649</td>
<td>0.1195</td>
<td>0.279</td>
</tr>
<tr>
<td></td>
<td>ITSE</td>
<td>1.4250</td>
<td>1.4025</td>
<td>0.8732</td>
<td><strong>22.9</strong></td>
<td>0.585</td>
<td>0.1032</td>
<td>0.235</td>
</tr>
<tr>
<td>1.0</td>
<td>IAE</td>
<td>1.9105</td>
<td>1.3435</td>
<td>0.8359</td>
<td>30.4</td>
<td><strong>0.559</strong></td>
<td>0.0952</td>
<td>0.232</td>
</tr>
<tr>
<td></td>
<td>ISE</td>
<td>1.1383</td>
<td>2.0000</td>
<td>1.4091</td>
<td>36.8</td>
<td>0.747</td>
<td><strong>0.0694</strong></td>
<td><strong>0.175</strong></td>
</tr>
<tr>
<td></td>
<td>ITAE</td>
<td>1.3029</td>
<td>0.9045</td>
<td>0.4269</td>
<td><strong>17.8</strong></td>
<td>0.687</td>
<td>0.1530</td>
<td>0.336</td>
</tr>
<tr>
<td></td>
<td>ITSE</td>
<td>1.4379</td>
<td>1.2208</td>
<td>0.7363</td>
<td>23.5</td>
<td>0.597</td>
<td>0.1070</td>
<td>0.239</td>
</tr>
</tbody>
</table>