Modelling of a cascade refrigeration system using artificial neural network

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SUMMARY

This study investigates the applicability of artificial neural networks (ANNs) to predict various performance parameters of a cascade vapour compression refrigeration system. For this aim, an experimental cascade system was set up and tested in steady-state operating conditions. Then, utilizing some of the experimental data for training, an ANN model for the system based on the standard back propagation algorithm was developed. The ANN was used for predicting the evaporating temperature in the lower-temperature circuit, compressor power for the lower and higher circuits and coefficients of performance for both the lower circuit and the overall cascade system. Afterwards, the performances of the ANN predictions were tested using new experimental data. The ANN predictions usually agreed well with the experimental results with correlation coefficients in the range of 0.953–0.996 and mean relative errors in the range of 0.2–6.0%. Furthermore, the ANN yielded acceptable predictions for the system performance outside the range of the experiments. The results suggest that the ANN approach can alternatively and reliably be used for modelling cascade refrigeration systems. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS: artificial neural networks; cascade refrigeration; R134a; coefficient of performance

1. INTRODUCTION

A vapour compression refrigeration system employing two or more thermally coupled refrigeration circuits, which operate at different pressure and temperature levels is called a cascade refrigeration system. Because high ratios of pressure across the compressor cause elevated discharge temperatures, low volumetric efficiencies and excessive stresses on compressor parts, the maximum allowable pressure ratio for reciprocating compressors is limited to about 9 (ASHRAE, 1990). As the pressure ratio tends to exceed this limit, it can be decreased by arranging the refrigeration system as a cascade one. The duty of the

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lower-temperature circuit in a cascade system is to provide the desired refrigeration effect at a relatively low temperature. Therefore, the condenser in this circuit is thermally coupled to the evaporator in the higher-temperature circuit. Thus, the evaporator in the higher circuit only serves to extract the heat released by the condenser in the lower one. The lower and higher circuits may use different refrigerants and refrigeration oils.

Although cascade refrigeration systems can be considered when an evaporating temperature below $-18^\circ C$ is desired (Dossat, 1991), they are usually employed to provide evaporating temperatures as low as $-100^\circ C$, which are generally required for some industrial processes involving liquefaction of gases. A cascade system is more expensive to build and more complicated than a single-stage system since it requires at least two refrigeration circuits.


This brief literature review reveals that investigators have usually preferred performing thorough and expensive experimental studies to determine various performance parameters of cascade refrigeration systems. Generally, cascade systems have not been modelled using classical techniques since the computer simulations are usually complicated and time consuming due to their dealing with the solution of complex differential equations. Moreover, the mathematical models require a large number of geometrical parameters defining the system, which may not be readily available. As an alternative to experimental investigation and mathematical modelling approaches, cascade systems can be modelled using artificial neural networks (ANNs) with significantly less engineering effort. Based on imitating the structure and mechanisms of the human brain, the ANNs allow modelling of the physical phenomena in complex systems without requiring explicit mathematical representations. Therefore, they can be applied to various engineering problems where classical approaches fail or they are too complicated to be used.


In this study, the ANN approach has been applied to a cascade vapour compression refrigeration system using R134a in both higher and lower circuits. After evaluating data
obtained from steady-state test runs of the experimental system, an ANN model for the system
has been developed. This model has been used for predicting various performance parameters of
the system, namely the evaporating temperature in the lower circuit, compressor power in both
circuits, and the coefficient of performance for both the lower circuit and the overall system.

2. ARTIFICIAL NEURAL NETWORKS

ANNs try to imitate the brain functions in a computerized way by resorting to the learning
mechanism as the basis of human behaviour. Learning by the samples from the experiments,
ANNs can be applied to the problems with no algorithmic solutions or with too complex
algorithmic solutions to be found. Therefore, ANNs are more flexible and powerful than the
parametric approaches (Hagan et al., 1996, Perlovsky, 2001).

An ANN consists of massively interconnected processing nodes called neurons. Each neuron
receives a weighted set of inputs and responds with an output. For this aim, the neuron first
forms the sum of the weighted inputs described as (Haykin, 1994)

$$n = \sum_{i=1}^{P} w_i x_i + b$$

where $P$ and $w_i$ are the number of elements and the interconnection weight of the input vector $x_i$,
respectively, and $b$ is the bias for the neuron. Then, the sum of the weighted inputs with a bias is
processed through an activation function, represented by $f$, and the output of this function is

$$f(n) = f\left(\sum_{i=1}^{P} w_i x_i + b\right)$$

The neural model mirrors the biological neuron that fires when its inputs are significantly
excited, i.e. the output of $f$ is large enough. The activation function can be defined in various
forms such as threshold function, sigmoid function, and hyperbolic tangent function.

ANNs are trained to reach from a particular input to a specific target output using a suitable
learning method. In the training process, ANNs perform a particular function by adjusting
interconnection weights between the processing nodes. This process goes on until the error
between the network output and desired output falls below a predetermined value or the
maximum number of epochs is exceeded. In the training process, the error is minimized by
modifying the weights and biases. Afterwards, the trained ANN can be used for predicting the
outputs for the inputs, which have not been introduced in the training process.

The architecture of an ANN usually consists of three parts, namely an input layer, one or
more hidden layers and an output layer. Each layer may have various numbers of neurons. The
information contained in the input layer, i.e. the values of the input parameters, is sent to
the output layer through the hidden layer(s). Each neuron can receive its input only from the
neurons in the lower layer and send its output to the neurons in the higher layer.

The performance of the predictions performed by an ANN is evaluated by a regression
analysis between the predicted parameters and the corresponding experimental values. This
evaluation is usually based on three criteria, namely the correlation coefficient, mean relative
error (MRE) and root mean square error (RMSE). The correlation coefficient assesses the
strength of the relationship between the predicted and experimental results. This coefficient
between \(a\) and \(p\) sets, that refer to the actual (experimental) output and predicted output sets, respectively, is defined as (Looney, 1997)

\[
R(a, p) = \frac{\text{Cov}(a, p)}{\sqrt{\text{Cov}(a, a)\text{Cov}(p, p)}}
\]

(3)

where \(\text{Cov}(a, p)\) is covariance between \(a\) and \(p\) sets given by (Looney, 1997)

\[
\text{Cov}(a, p) = E[(a - \mu_a)(p - \mu_p)]
\]

(4)

where \(E\) is the expected value, \(\mu_a\) is the mean value of \(a\) set and \(\mu_p\) is the mean value of \(p\) set. If \(\text{Cov}(a, p) = 0\), then \(a\) and \(p\) are said to be uncorrelated. Likewise, \(\text{Cov}(a, a)\) and \(\text{Cov}(p, p)\) are the auto covariances of \(a\) and \(p\) sets, correspondingly, and given by

\[
\text{Cov}(a, a) = E[(a - \mu_a)^2]
\]

(5)

\[
\text{Cov}(p, p) = E[(p - \mu_p)^2]
\]

(6)

The correlation coefficient can range between \(-1\) and \(+1\). \(R\) values closer to \(+1\) indicate a stronger positive linear relationship, while \(R\) values closer to \(-1\) indicate a stronger negative relationship.

The mean ratio between the errors and experimental values is indicated by the MRE defined as

\[
\text{MRE}(\%) = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{100 (a_i - p_i)}{a_i} \right|
\]

(7)

where \(N\) is the number of the points in the data set.

Finally, the RMSE is evaluated by

\[
\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (a_i - p_i)^2}
\]

(8)

### 3. DESCRIPTION OF THE EXPERIMENTAL SET-UP

The ANN modelling has been applied to the cascade vapour compression refrigeration system shown in Figure 1. The system has a lower-temperature and a higher-temperature circuit coupled to each other by means of a water stream. The lower circuit consists of a reciprocating compressor, a shell-and-coil-type water-cooled condenser, a liquid receiver, a filter-drier, a thermostatic expansion valve and an electrically heated evaporator. The twin-cylinder open-type compressor in the lower-circuit has a swept volume of 75.7 cm\(^3\) rev\(^{-1}\) and a nominal speed of 460 rpm. It was belt-driven by a single-phase electric motor. The water-cooled condenser has a heat transfer area of 0.075 m\(^2\), and consists of a vertical coil enclosed in welded steel shell. The evaporator was made from copper tube and has two electric heaters rolled inside the tube. The desired refrigeration load was applied to the evaporator by adjusting the voltage across the electric heaters via a variable transformer. The lower-temperature circuit was charged with 600 g of R134a.

The higher-temperature refrigeration circuit linked to the lower-temperature one consists of a hermetic compressor, an air-cooled condenser, a liquid receiver, a filter-drier, a thermostatic
expansion valve and a tube-in-tube evaporator. The compressor in this circuit has a swept volume of 8.85 cm$^3$ rev$^{-1}$ and a nominal speed of 2800 rpm. The condenser was made from aluminium-finned copper tubing. The higher-temperature circuit was charged with 750 g of R134a.

To perform cascade operation, the water-cooled condenser in the lower circuit was thermally coupled to the evaporator in the higher circuit by means of a refrigerated water stream. The water circuit between these components consists of a circulation pump, a water tank, plastic tubing and a hand-operated valve controlling the water flow rate in the circuit. All components in the refrigeration and water circuits and the pipelines were insulated with either polyurethane foam or elastomeric insulator. The entire cascade system was located in an air-conditioned room where air dry bulb temperature could be maintained at the desired value.

Figure 1 also shows the locations where mechanical and electrical measurements were conducted. Mechanical measurements consists of temperature, pressure and mass flow rate measurements performed on both refrigeration circuits including the water circuit, while electrical measurements are the voltage across the electrical heaters and current flow through the heaters in the lower-circuit evaporator.

All temperature measurements were performed utilizing K-type thermocouples. Thermocouples for refrigerant temperature were located at the inlet and outlet of each component in both refrigeration circuits and soldered to the copper tube, while thermocouples for water were in direct contact with the water stream. The suction and discharge line pressures in both circuits were measured by Bourdon tube gages. Because the refrigeration lines are relatively short, the evaporator and condenser pressures were assumed to be equal to the measured ones. The refrigerant mass flow rates were measured by individual variable-area flow meters located in the liquid line of each refrigeration circuit. The mass flow rate of the water stream through the water-cooled condenser was measured by another variable-area flow meter with a needle control valve. Some features of the instrumentation are summarized in Table I.
4. THERMODYNAMIC ANALYSIS OF THE EXPERIMENTAL SYSTEM

The evaporator load on the lower circuit, i.e. the refrigeration capacity of the cascade system, can be evaluated for both the refrigerant and the heaters sides. Assuming that the evaporator was insulated perfectly, the evaporator loads for both sides can be equated:

\[ Q_e = \dot{m}_{r1}(h_{\text{e1},o} - h_{\text{e1},i}) \approx VI \]  \hspace{1cm} (9)

As seen in Equation (9), the evaporator load for the heaters side is based on the results of voltage and current measurements, while that for the refrigerant side relies on the measurements of the refrigerant mass flow rate, pressure and temperature at the outlet and inlet of the evaporator. The evaporator load deviations between two sides were usually within \( \pm 5\% \), and only the heaters side results were used as the evaporator load due to their yielding lower uncertainties, as indicated by the uncertainty analysis presented in the next section. Thus, the refrigerant mass flow rate in the lower circuit based on the evaporator load for the heaters side can be evaluated from

\[ \dot{m}_{r1} = \frac{VI}{h_{\text{e1},o} - h_{\text{e1},i}} \]  \hspace{1cm} (10)

The accuracy for the refrigerant mass flow rate measured by the variable-area flow meter was equal to \( \pm 5\% \), which was poorer than the uncertainty for the flow rates determined from Equation (10). Hence, only the results of this equation were used as the refrigerant mass flow rate in the lower circuit and the results of direct measurements were used for only checking purposes.

Assuming that the compression process is adiabatic, the compressor power in the lower circuit can be expressed as

\[ W_{c1} = \dot{m}_{r1}(h_{\text{c1},o} - h_{\text{c1},i}) \]  \hspace{1cm} (11)

The coefficient of performance of the lower circuit can be evaluated from the ratio of the refrigeration capacity to the compressor power:

\[ \text{COP}_1 = \frac{Q_e}{W_{c1}} \]  \hspace{1cm} (12)

Assuming that both the condenser in the lower circuit and the evaporator in the higher circuit were perfectly insulated and the system operates in steady state, the heat rejected by the refrigerant in the lower circuit condenser can be related to the heat absorbed by the refrigerant in the higher-circuit evaporator as follows:

\[ \dot{m}_{r1}(h_{\text{con1},i} - h_{\text{con1},o}) + |W_p| + Q_{\text{gain}} \approx \dot{m}_{r2}(h_{\text{e2},o} - h_{\text{e2},i}) \]  \hspace{1cm} (13)
where \( |W_p| \) is the power absorbed by water in the circulation pump, \( Q_{\text{gain}} \) is the heat gain through the components of the water circuit and \( \dot{m}_{r2} \) is the refrigerant mass flow rate in the higher circuit measured by the flow meter. The heat gain through the water circuit is directly proportional to the difference between the ambient air and water temperatures. On the other hand, the water temperature at the outlet of the higher evaporator decreases with decreasing water flow rates. Therefore, the lower the water flow rate, the higher the heat gain through the water circuit is. Consequently, the evaporator load in the higher circuit increases with decreasing water flow rate. Furthermore, the water flow rate also influences the condensing temperature in the lower circuit and the evaporating temperature in the higher circuit, thereby affecting the compressor power in both circuits.

Assuming that the compression process in the higher circuit is also adiabatic, the compressor power in the higher circuit can be determined from

\[
W_{c2} = \dot{m}_{r2}(h_{c2,o} - h_{c2,i})
\]  

Finally, the energetic performance of the cascade refrigeration system can be evaluated by the overall coefficient of performance defined as

\[
\text{COP}_{\text{cas}} = \frac{Q_e}{W_{c1} + W_{c2}}
\]

5. TESTING PROCEDURE

In the experimental study, totally 24 different steady-state test runs were conducted to gather data for developing the proposed ANN. The inputs varied in the tests were the evaporator load and the flow rate of the water stream circulated between the refrigeration circuits. The evaporator load was kept constant at 248, 357, 486, 636, 805 and 995 W. For each evaporator load, the water flow rate was kept constant at 8, 12, 18 and 26 g s\(^{-1}\). During the tests, the air dry bulb temperature inside the room containing the experimental cascade system was maintained at 24.0 ± 0.5°C. Based on the data acquired in the test operations, various performance parameters of the cascade system were evaluated from the equations presented in the previous section. The values for these parameters were used for both training the proposed ANN and testing its performance.

6. UNCERTAINTY ANALYSIS

Uncertainty analysis for the calculated parameters, namely \( Q_e, \dot{m}_{r1}, W_{c1}, W_{c2}, \text{COP}_1 \) and \( \text{COP}_{\text{cas}} \) were performed using the method given by Moffat (1988). This method first assumes that the function \( Z \) is calculated from a set of totally \( M \) measurements (independent variables) represented by

\[
Z = Z(Y_1, Y_2, Y_3, \ldots, Y_M)
\]

Then, the uncertainty of the result \( Z \) can be evaluated by combining uncertainties of individual terms using a root-sum-square method:

\[
\delta Z = \left\{ \sum_{i=1}^{M} \left( \frac{\partial Z}{\partial Y_i} \delta Y_i \right)^2 \right\}^{1/2}
\]
Using accuracies for various measured variables presented in Table I, uncertainties of the calculated parameters were determined with the evaluation of Equations (9)–(15) in Equation (17). The uncertainties of $Q_e$, $m_{r1}$, $W_{c1}$, $W_{c2}$, COP$_1$ and COP$_{cas}$ estimated by the analysis are 4.4, 4.4, 16.2, 16.5, 16.8 and 7.9%, respectively.

7. MODELLING WITH THE ANN

The ANN model for the experimental cascade refrigeration system was developed using the available data set from the experimental work, which consists of totally 24 input–output pairs. A total of 70% of the data set, corresponding to the results of 17 test runs, was randomly assigned as the training set, while the remaining 30% was employed for testing the performance of the ANN predictions.

The architecture of the ANN for the cascade refrigeration system is schematically shown in Figure 2. The inputs to the network are the evaporator load ($Q_e$) and water mass flow rate ($\dot{m}_w$), while the outputs are the evaporating temperature ($T_e$), compressor power in the lower circuit ($W_{c1}$), coefficient of performance for the lower circuit (COP$_1$), compressor power in the higher circuit ($W_{c2}$) and the overall coefficient of performance for the cascade refrigeration system (COP$_{cas}$). Therefore, the ANN consists of an input layer with two neurons and an output layer with five neurons.

The accuracy of ANN predictions considerably depends on the number of hidden layers and the number of neurons in each hidden layer. After trying various network configurations, the network was decided to have only one hidden layer with four neurons. Therefore, the

![Figure 2. The structure of the ANN for modelling the cascade refrigeration system.](image)
predictions for the five output parameters were performed using a three layer feed forward ANN. The activation function in the hidden layer, a network characteristic affecting the network performance, was chosen as tangent sigmoid function. In order to train the ANN, the input vectors with two variables and the corresponding target vectors with five variables from the training set were introduced to the network utilizing standard back-propagation algorithm (Hagan, 1996), a widely used algorithm in engineering applications. In the training procedure, the weighting coefficients were adjusted using Levenberg–Marquardt algorithm, which is a variant of the back-propagation algorithm. The output of the ANN was compared to the desired output, i.e. target, at each presentation, and an error was computed. Then, this error was back-propagated to the network, which in turn used for adjusting the weights in such a way that each iteration results in a lower error and the ANN output progressively approximates to the desired output. As a function between the network inputs and outputs was approximated, the training procedure was terminated. Finally, the input vectors from the test data set were introduced to the ANN, and the responses of the network to the input vectors, i.e. the ANN predictions, were compared with the experimental results. The computer code used for solving the back-propagation algorithm and testing the ANN performance was implemented under the MATLAB environment.

8. RESULTS AND DISCUSSION

Figures 3–5 indicate the predictions of the trained ANN for various performance parameters of the experimental cascade refrigeration system versus the experimental (actual) values. The comparisons in these figures were based on data from the test set, which was not introduced to the ANN in the training procedure. Figures 3 and 4 are provided with a straight line indicating perfect prediction and an error band, both used for evaluating the performance of the ANN predictions.

![Figure 3](image_url)

Figure 3. The ANN predictions for the evaporating temperature in the lower refrigeration circuit vs the experimental values.
Figure 3 shows the ANN predictions for the evaporating temperature in the lower refrigeration circuit as a function of the experimental values. The ANN predictions for $T_e$ yield a MRE of 0.2%, a RMSE of 0.9 K and a correlation coefficient of 0.996 with the experimental values.

Figure 4. The ANN predictions for the overall coefficient of performance of the cascade refrigeration system vs the experimental values.

Figure 5. Comparisons of the ANN predictions and experimental results for various performance parameters of the cascade refrigeration system.

Figure 3 shows the ANN predictions for the evaporating temperature in the lower refrigeration circuit as a function of the experimental values. The ANN predictions for $T_e$ yield a MRE of 0.2%, a RMSE of 0.9 K and a correlation coefficient of 0.996 with the experimental values.
data. Because the temperature of a refrigerated space considerably depends on the evaporating
temperature, the ANN predicting $T_e$ remarkably would also perform well in predicting the
temperature of the refrigerated space in a more realistic application where the evaporator was
located inside the refrigerated space.

The ANN predictions for the overall coefficient of performance of the cascade refrigeration
system as a function of the experimental values are depicted in Figure 4. For this performance
parameter, the ANN predictions yield a MRE of 6.0%, a RMSE of 0.1 and a correlation
coefficient of 0.953, which are poorer than the previous results. This poor performance is due to
the fact that the COP$_{cas}$ depends on the performance evaluations of three different components,
namely the evaporator and compressor in the lower circuit, and the compressor in the higher
circuit. As revealed by the uncertainty analysis presented previously, these components
introduce three sources of uncertainty to the experimental results, thus causing an imperfect
training and impairing the performance of the ANN predictions.

The performances of the ANN predictions for $T_e$, $W_{c1}$, COP$_1$, $W_{c2}$ and COP$_{cas}$ are indicated
in Figure 5 in an alternative form. It is seen that the test patterns consist of 7 tests, corresponding to about 30% of 24 tests. On the other hand, the statistical values of the network
predictions for all output parameters are reported in Table II. It is clear that the ANN predicts
$T_e$, $W_{c1}$ and COP$_1$ excellently while predictions for $W_{c2}$ and COP$_{cas}$ are slightly poorer. The
Poor performance of $W_{c2}$ predictions probably originates from the fact that the refrigerant mass
flow rate in the higher circuit was measured directly using a variable-area flow meter within an
accuracy of $\pm$ 5%. Because the refrigerant flow rate in the lower circuit was evaluated from the
evaporator load for the heaters side, it has a better uncertainty. The compressor power is a
function of the refrigerant mass flow rate. Therefore, $W_{c1}$ values introduced to the network
during the training are more accurate than $W_{c2}$ values. Consequently, the ANN predictions for
$W_{c2}$ are poorer than those for $W_{c1}$. However, if a higher number of test runs had been
performed to provide a larger amount of experimental data for training the ANN, better
network predictions would have been expected.

Figure 6 shows the ANN predictions for the evaporating temperature and overall coefficient
of performance of the experimental system with respect to evaporator load for a constant water
flow rate of 15 g s$^{-1}$. Figure 6(a) reports the predictions in the range of the evaporator loads
employed in the experiments, while Figure 6(b) indicates those beyond the range of the
experiments. Although the accuracies of the network predictions in both cases were not
measured, it is seen that the ANN yields agreeable curves for the predicted parameters,
especially in the range of the experiments. On the other hand, it is clear that both the
evaporating temperature and overall coefficient of performance increase with increasing
evaporator load for a constant water flow rate. An increase in the evaporator load causes

<table>
<thead>
<tr>
<th>Outputs</th>
<th>$R$ (training)</th>
<th>MRE% (training)</th>
<th>RMSE (training)</th>
<th>$R$ (test)</th>
<th>MRE% (test)</th>
<th>RMSE (test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_e$ (K)</td>
<td>0.998</td>
<td>0.2</td>
<td>0.6</td>
<td>0.996</td>
<td>0.2</td>
<td>0.9</td>
</tr>
<tr>
<td>$W_{c1}$ (W)</td>
<td>0.998</td>
<td>1.8</td>
<td>2.5</td>
<td>0.994</td>
<td>3.6</td>
<td>4.7</td>
</tr>
<tr>
<td>COP$_1$</td>
<td>0.996</td>
<td>1.2</td>
<td>0.1</td>
<td>0.970</td>
<td>3.6</td>
<td>0.2</td>
</tr>
<tr>
<td>$W_{c2}$ (W)</td>
<td>0.990</td>
<td>2.2</td>
<td>6.6</td>
<td>0.985</td>
<td>3.9</td>
<td>9.9</td>
</tr>
<tr>
<td>COP$_{cas}$</td>
<td>0.993</td>
<td>2.9</td>
<td>0.1</td>
<td>0.953</td>
<td>6.0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table II. Statistical values of predictions.

lesser increases in the compressor power for both the lower and higher circuits, thus yielding a higher COP$_{cas}$.

Figure 7 indicates ANN predictions for the evaporating temperature and overall coefficient of performance with respect to water flow rate for a constant evaporator load of 700 W.

Figure 6. ANN predictions for the evaporating temperature and overall coefficient of performance as a function of evaporator load for $\dot{m}_w = 15$ g s$^{-1}$: (a) predictions within the range of the experiments; and (b) predictions beyond the range of the experiments.

lesser increases in the compressor power for both the lower and higher circuits, thus yielding a higher COP$_{cas}$.

Figure 7 indicates ANN predictions for the evaporating temperature and overall coefficient of performance with respect to water flow rate for a constant evaporator load of 700 W.
Figure 7(a) shows the predictions in the range of the water flow rates employed in the experiments, while Figure 7(b) reports those beyond the range of the experiments. The accuracies of the network predictions in both cases were also not measured. However, it is seen that the ANN yields satisfying curves for the predicted parameters both within and beyond the experimental range. For a constant evaporator capacity, the evaporating temperature usually
decreases slightly with increasing water flow rate, while the water flow rate does not have a significant effect on the overall COP, especially within the range of the experiments.

9. CONCLUSIONS

The ANN approach has been applied to the modelling of a cascade vapour compression refrigeration system using R134a in both lower and higher-temperature refrigeration circuits. Data for training and testing the proposed ANN was obtained from the steady-state test runs of an experimental cascade refrigeration system. After developing the ANN based on back-propagation algorithm, it was used for predicting the evaporating temperature in the lower circuit, compressor power for each circuit, COP for the lower circuit and COP for the overall cascade system. The performances of the ANN predictions were determined in terms of the correlation coefficient, mean relative error and root mean square error. The ANN predictions for the cascade refrigeration system usually yielded a good statistical performance with correlation coefficients in the range of 0.953–0.996 and MREs in the range of 0.2–6.0% along with extremely low RMSE values compared to the ranges of the experimental results. The ANN was also used for predicting the system performance outside the range of the experiments, and acceptable prediction curves were obtained.

This study shows that, instead of classical modelling techniques, the ANN approach can be employed for modelling cascade vapour compression refrigeration systems. Consequently, performance parameters of these systems can easily be determined by performing only a limited number of test runs instead of conducting an exhaustive experimental study or dealing with a complicated mathematical model, thus saving engineering effort and funds.

NOMENCLATURE

\[ a = \text{actual output} \]
\[ \text{ANN} = \text{artificial neural network} \]
\[ b = \text{bias} \]
\[ \text{COP} = \text{coefficient of performance} \]
\[ \text{Cov} = \text{covariance} \]
\[ E = \text{expected value} \]
\[ f = \text{activation function} \]
\[ h = \text{enthalpy of the refrigerant (kJ kg}^{-1}\text{)} \]
\[ I = \text{current flow through the heaters (A)} \]
\[ m = \text{mass flow rate (g s}^{-1}\text{)} \]
\[ M = \text{total number of independent variables in function } Z \]
\[ \text{MRE} = \text{mean relative error} \]
\[ n = \text{sum of the weighted inputs} \]
\[ N = \text{number of the points in the data set} \]
\[ p = \text{predicted output} \]
\[ P = \text{number of elements in the input vector} \]
\[ Q_e = \text{refrigeration capacity of the cascade system (W)} \]
\[ Q_{\text{gain}} = \text{heat gain through the components of the water circuit (W)} \]
\( R \) = correlation coefficient
RMSE = root mean square error
\( T_e \) = evaporating temperature in the lower-temperature circuit (K)
\( V \) = voltage across the heaters (V)
w = interconnection weight
\( W_c \) = compressor power (W)
\( |W_p| \) = power absorbed by water in the circulation pump (W)
\( x \) = input of a neuron
\( Y \) = independent variable
\( Z \) = a function of independent variables

**Greek symbols**

\( \mu_a \) = mean value of set \( a \)
\( \mu_p \) = mean value of set \( p \)

**Subscripts**

c = compressor
cas = cascade
con = condenser
e = evaporator
i = inlet
o = outlet
r = refrigerant
w = water
l = lower-temperature circuit
2 = higher-temperature circuit

**REFERENCES**


