Numerical investigation of flow and heat transfer in an inclined cavity with wavy walls

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Abstract: In this study, natural heat transfer in an inclined cavity with two isothermal wavy walls and two adiabatic straight walls is investigated numerically. A parametric study is carried out for Rayleigh numbers of $10^4$, $10^5$ and $10^6$, for the inclination angle of $0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, $150^\circ$, and $180^\circ$ and for the undulation numbers of one, two and three. Computations show that the flow structure in the cavity with wavy walls is affected significantly with undulation number. The mean Nusselt number reduces when undulation number increases. The effect of undulation number on the mean Nusselt number is more pronounced at high Rayleigh numbers. The variation of mean Nusselt numbers with inclination angle shows maxima around $90^\circ$ of inclination angle. It is shown that the amount of heat transfer rate in the cavity can be increased up to 15% by introducing wavy walls for inclination angle of smaller than $90^\circ$, comparing with square cavity. The effectiveness of wavy walls diminishes when inclination angle is greater than $90^\circ$. The amount of heat transfer decreases when the number of undulations is increased from two to three.

Keywords: Natural heat transfer, inclined cavity, wavy wall

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INTRODUCTION

Natural convection inside enclosures has been studied extensively by researchers since it is frequently encountered in many engineering applications such as solar collectors, cooling of electronic equipments etc. The studies about natural convection inside enclosures are mostly focused on rectangular enclosures considering different Rayleigh number, inclination angle and aspect ratio [1-4 to cite just few].

Recently, natural convection in wavy enclosures has attracted considerable attention due to its importance in geophysical applications and heat exchangers design [5]. Natural convection heat transfer along a vertical wavy surface is investigated theoretically by Yao [6,7] and concluded that heat transfer rate per unit-wetted wavy surface is smaller than that of the corresponding flat plate while the total heat-transfer rate for a wavy surface is greater than that of a flat plate. It is also shown in [6,7] that the enhanced total heat-transfer rate seems to depend on the ratio of amplitude and wavelength of surface. Heat transfer and fluid characteristics inside an enclosure by two isothermal wavy walls and two adiabatic straight walls was investigated numerically in [8]. Higher heat transfer was observed at lower aspect ratios for a certain Grashof number which depends strongly on the surface waviness. Laminar natural convection in an inclined square cavity with a wavy wall for one and three undulations was investigated numerically in [9] for different Rayleigh numbers, inclination angles. It was shown in the study that the mean Nusselt number decreases comparing with that of square cavity and an increase in the undulation number on the hot wall reduces the heat transfer rate for an inclination angle greater than 75°. The effect of non uniform boundary condition and aspect ratio on the natural heat transfer in wavy rectangular enclosures is investigated in [10, 11]. It was reported that the mean Nusselt number decreases comparing with that of the square cavity and increasing aspect ratio results in a decrease in the Nusselt number. Laminar natural convection in an enclosure with three flat walls and one wavy wall for one-, two- and three-undulations is investigated in [12] for different Rayleigh numbers, inclination angles and amplitudes. A sinusoidal temperature profiles was specified in a flat wall while other three walls are maintained at constant cold temperature. The results obtained in [12] showed that the angle of inclination affects the flow and heat transfer rate in the cavity. With increase in amplitude, the average Nusselt number on the wavy wall is appreciably high at low Rayleigh number. Increasing the number of undulations beyond two is not beneficial. The trend of local Nusselt number is wavy [12]. Varol and Oztop investigated natural convection heat transfer inside a wavy walled and inclined solar collector and compared
with a flat collector in [13,14]. They reported that more heat transfer is obtained in the case of wavy collector than that of flat one, Nusselt number decreases with the inclination angle and the highest Rayleigh number is obtained at the highest inclination angle which is on contrary to flat collectors.

In this study, the flow structure and heat transfer in a cavity bounded by two isothermal wavy walls and two adiabatic straight walls is investigated numerically. A parametric study is carried out for different Rayleigh number, inclination angle and undulation number.

2 PROBLEM DESCRIPTION AND NUMERICAL METHOD

The geometry of interest is depicted in Fig. 1 which is a closed square cavity with two wavy and two straight walls. The height of walls is H. Curvature of the wavy walls is defined as:

\[ f(y) = [0.95 + 0.05(\cos2\pi ny)] \]  

where \( n \) is undulation number.

Computations are carried out for seven different inclination angle (\( \theta \)) which is varied in the range of 0° and 180°. The flow in the cavity is considered to be two dimensional, steady, laminar and incompressible. The fluid in the cavity is air and whose properties are assumed to be constant. Viscous dissipation is neglected and buoyancy force is treated by boussinesq approximation.

![Fig. 1 Schematic representation of cavity with two wavy walls.](image)

The incompressible forms of the governing equations in Cartesian co-ordinates for laminar flow are:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + g\beta(T - T_\infty)\sin\theta \]
\[
\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} \\
+ v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\
+ g\beta (T - T_\infty) \cos \theta
\]  
\[ (4) \]

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]
\[ (5) \]

These equations are solved assuming with no-slip boundary conditions on the solid surfaces. Fixed temperature is applied on the wavy walls and straight walls are assumed to be adiabatic.

The computed results are nondimensionalized by Rayleigh number and Nusselt number which are defined in Eq. (6) and Eq.(7), respectively.

\[
Ra = \frac{g\beta \Delta T H^3}{v^2} Pr
\]
\[ (6) \]

Here, \( Pr \) is Prandtl number which is assumed to be 0.71. The local Nusselt number is calculated along the hot wall as:

\[
Nu_x = - \frac{\partial T}{\partial y}
\]
\[ (7) \]

The calculation of mean Nusselt number is given as:

\[
Nu = \int_0^1 Nu_x \, dy
\]
\[ (8) \]

A grid independency study is performed to finalize the grid points for further investigation. Therefore, several grid points namely 21x21, 41x41, 81x81, 161x161 are tested. Results showed that a grid independent solution is achieved with 81x81 grid points. The computed results are validated by comparing results with [3] for different Rayleigh number.

3 RESULTS AND DISCUSSION

The variation of both streamline and temperature contours with inclination angle for cavity with one undulation and Ra=10^6 is given in Fig. 2. Also, the effect of inclination angle on the local Nusselt number is given in Fig. 3. For \( \theta=0^\circ \) two counter-rotating rolls occur on both hot and cold surfaces even with low fluid motion. Fluid is almost stationary between these rolls. As seen in Fig. 2a-2, thermal stratification appears in the cavity due to weak motion of fluid. Consequently, heat is transferred from top surface to bottom surface by conduction. Although fluid motion has not significant effect on the temperature variation, it hits the crest and causes a small increase on the Nusselt number near \( y/H=0.5 \) as seen in Fig. 3.

For \( \theta=30^\circ \), fluid is driven upward near the hot surface and driven downward near the cold surface due to buoyancy. Thus, as seen in Fig. 2c-1 three circulation cells occur in the cavity and all of them rotate in the same direction. The primary cell circulates all over the cavity and the secondary cells which cover the half of the cavity occur on both hot and cold surface sides. In addition, a buffer region occurs between these secondary cells. As seen in the Fig. 2b-2 isotherms still stay horizontal which indicates that thermal stratification is still valid, however thermal boundary layer increases in the flow direction which gradually reduces the Nusselt number as \( y/H \) decreases.
For $\theta=60^\circ$ the cavity is almost covered with a single primary circulation cell and the secondary cells appeared for $\theta=30^\circ$ nearly diminish. While isotherms are still horizontal in the middle of the cavity they are quasi-parallel to surfaces near the cold and hot walls. Comparing with for $\theta=30^\circ$, the local Nusselt number for $\theta=60^\circ$ increases significantly on the hot surface. Again thermal boundary layer increases in the flow direction which reduces the Nusselt number significantly as $y/H$ decreases.

For $\theta=90^\circ$ the buoyancy accelerates the flow and creates only a single primary circulation cell. The secondary cells appeared for $\theta=30^\circ$ completely diminishes here. Isotherms are horizontal in the middle of the cavity. As seen in the Fig. 3, the local Nusselt number for $\theta=90^\circ$ reaches the maximum value. Again thermal boundary layer increases in the flow direction which reduces significantly the local Nusselt number as $y/H$ decreases.

**Fig. 2** Streamline (1) and temperature (2) distributions for cavity with one undulation and for $Ra=10^6$ a) $\theta=0^\circ$ b) $\theta=30^\circ$ c) $\theta=60^\circ$ d) $\theta=90^\circ$ e) $\theta=120^\circ$ f) $\theta=150^\circ$ g) $\theta=180^\circ$
For $\theta=120^\circ$ three circulation cells occur in the cavity as similar to the case $\theta=30^\circ$. The primary cell covers the whole cavity and two secondary cells appear in the primary cell. The hot surface at the right side causes higher buoyancy which accelerates fluid and causes higher Nusselt number than the case for $\theta=30^\circ$ (Fig. 3).

For $\theta=150^\circ$ and $\theta=180^\circ$, horizontal isotherms thus thermal stratifications disappear. While a single circulation cell is present in the whole cavity for $\theta=150^\circ$, secondary circulation cells occur around the corners for $\theta=180^\circ$. The variation of local Nusselt number of these cases shows different behavior than for the cases for $\theta<150^\circ$. The maximum Nusselt number occurs in the vicinity of the crest. The cells near the corners hold the fluid and increase the thickness of thermal layer around the corners that cause decrease in the heat transfer rate thus reduces the Nusselt number around this region.

![Fig. 3 Variation of local Nusselt number with inclination angle for cavity with one undulation and $Ra=10^6$](image)

The results shown in Fig. 2 are re-plotted for three undulations in Fig. 4. Generally speaking, the increase on the undulation number does not affect the flow structure much. The cavity is again covered with a primary circulation cell for the same inclination angles for the cavity with one undulation. Secondary circulation cells appear around the center of the cavity for $\theta=30^\circ$, 60$^\circ$ and 120$^\circ$ similar to Fig. 2. The main difference occur for $\theta=0^\circ$ where the number of counter-rotating rolls on each side increases from two to six.

The variation of local Nusselt number with inclination angle for $Ra=10^6$ is given in Fig. 5 and Fig. 6, for undulation numbers of two and three, respectively. Although the increase of undulation number has not a noticeable effect on the flow structure at first sight, it has significant effect on the Nusselt number. While the variation of Nusselt number shows a single considerable peak value in the vicinity of crest for only $\theta=180^\circ$ in the case of one undulation, nearly all cases for three undulations have a peak of Nusselt value near each crest. Fluid speeds up and slows down when passing over the crest which causes a wavy variation of local Nusselt number.
Fig. 4 Streamline (1) and temperature (2) distributions for cavity with three undulations and for $Ra=10^6$ a) $\theta=0^\circ$ b) $\theta=30^\circ$ c) $\theta=60^\circ$ d) $\theta=90^\circ$ e) $\theta=120^\circ$ f) $\theta=150^\circ$ g) $\theta=180^\circ$
The mean Nusselt numbers are obtained by Eq.(8). The effect of the undulation number, inclination angle and Rayleigh number on the mean Nusselt number is shown in Fig. 7. As seen in the Fig., the variation of Nusselt number with inclination is the same for all Rayleigh numbers. The mean Nusselt number is close to unity in the range of $\theta=0^\circ-30^\circ$ where hot wall is at the top and heat is transferred by conduction. Nusselt number increases until a certain inclination angle and then decreases. The inclination angle which maximizes the mean Nusselt number decreases as the Rayleigh number increases. As the Rayleigh number increases, the mean Nusselt number also increases as expected. Increasing the number of undulation reduces the mean Nusselt number for all cases. However, the effect of undulation number is more pronounced as Rayleigh number increases.
The amount of heat transfer in the cavity with wavy walls depends not only on mean Nusselt number but also the magnitude of heat transfer area. As undulation number increases, the mean Nusselt number reduces (as shown in Fig. 7), while heat transfer area increases by introducing wavy walls. The increase in the heat transfer area is 2.6%, 9.26% and 19.4% for one, two and three undulations, respectively, comparing with square cavity. In order to see the net effect of wavy walls on the amount of the heat transfer, the computed values of cavity with wavy walls are non-dimensionalized using the values obtained for square cavity ($Q/Q_{\text{square}}$) and presented in the following Fig.s. Fig. 8 shows the variations of $Q/Q_{\text{square}}$ with inclination angle and undulation number. As seen in Fig. 8, the magnitude of the heat transfer of wavy walls increases 15% for $\theta=0^\circ$. As $\theta$ increases, the effect of wavy wall decreases significantly until $\theta=90^\circ$. The effect of wavy wall on the heat transfer seems to be negligible where $Q/Q_{\text{square}}\approx 1$, for $\theta>90^\circ$. As seen in the Fig., the effectiveness of wavy wall increases as undulation number is increased from one to two. However, the effectiveness decreases as the undulation number is increased from two to three which is consistent with [6, 9, 12].

Fig. 7 Variation of mean Nusselt number with inclination angle, Rayleigh number and undulation number.

Fig. 8 Variation of $Q/Q_{\text{square}}$ with inclination angle and undulation number for $Ra=10^4$
The effect of Rayleigh number and inclination angle on $Q/Q_{\text{square}}$ is shown in Fig. 9 for the cavity with three undulations. The behavior of $Q/Q_{\text{square}}$ with inclination angle is similar to the results drawn above for Fig 8. As Rayleigh number increases, the effectiveness of wavy wall increases for $\theta<90^\circ$ and decreases as for $\theta\geq90^\circ$. For $Ra=10^6$ and $\theta\geq90^\circ$, $Q/Q_{\text{square}}$ is smaller than unity which shows that the effect of reduction of mean Nusselt number is more dominative than the increase of heat transfer area.

![Fig. 9 Variation of $Q/Q_{\text{square}}$ with inclination angle and Rayleigh number for cavity with three undulations.](image)

### 4 CONCLUSIONS

In this study, flow and heat transfer in an inclined cavity with two wavy walls are investigated numerically. The effect of undulation number, inclination angle and Rayleigh number is studied. Computations show that the flow structure in the cavity depends on these parameters. The cavity is covered with a primary circulation cell for all cases. Secondary circulation cells appear around center of the cavity for $\theta=30^\circ$, 60$^\circ$ and 120$^\circ$ which causes reductions on the local Nusselt number. Isotherms are horizontal and thermal stratification occurs in the cavity when inclination angle is less than 150$^\circ$. The variation of local Nusselt number is wavy. The mean Nusselt number reduces when undulation number increases. The effect of undulation number on the mean Nusselt number is more pronounced at high Rayleigh numbers. It is shown that the amount of heat transfer rate in the cavity can be increased up to 15% by introducing wavy walls. The effectiveness of wavy walls diminishes as the number of undulations is increased from two to three.

### REFERENCES

