τ → μνν̄ decay in the general two Higgs doublet model
We study $\tau \to \mu \bar{\nu}_i \nu_i$, $i = e, \mu, \tau$ decay in the model III version of the two Higgs doublet model. We calculated the $BR$ at the order of magnitude $10^{-6} \sim 10^{-4}$ for intermediate values of the Yukawa couplings. Furthermore, we predict the upper limit of the coupling for the $\tau \to h^0(A^0)\tau$ transition as $\sim 0.3$ in the case that the $BR$ is $\sim 10^{-6}$. We observe that the experimental result of the process under consideration can give comprehensive information about the physics beyond the standard model and the free parameters existing.

1. Introduction
Lepton flavour violating (LFV) interactions are interesting since they do not exist in the standard model (SM) and give a strong signal about the new physics beyond. Such decays have achieved great interest at present and the experimental search has been improved. Among LFV decays the ones existing in the leptonic sector are clean theoretically in the sense that they are free from non-perturbative effects. The processes $\mu \to e\gamma$, $\tau \to e(\mu)\gamma$, $\tau \to e\bar{e}e$, $\tau \to e\bar{\mu}\mu$ are examples of LFV interactions. There are on-going and planned experiments for $\mu \to e\gamma$ ($\tau \to \mu\gamma$) and the current limits for their branching ratios ($BR$) are $1.2 \times 10^{-11}$ [1] ($1.1 \times 10^{-6}$ [2]). The numerical estimates predict that the $BR$ of the processes $\tau \to e\bar{e}e$, $\tau \to e\bar{\mu}\mu$ are of the order of magnitude $10^{-6}$ [3], which is a measurable value in the LEP experiments and $\tau$ factories.

In such decays, the assumption of the non-existence of Cabibbo–Kobayashi–Maskawa (CKM) type matrix in the leptonic sector forbids the charged flavour changing (FC) interactions and, therefore, the physics beyond the SM plays a main role, where the general two Higgs doublet model (2HDM), so-called model III, is one of the candidates. This model predicts that the LFV interactions can exist at loop level and the internal neutral Higgs bosons $h_0$ and $A_0$ play a main role. There are a number of Yukawa couplings which describe the strength of the interactions lepton–lepton–neutral Higgs particle, appearing in the loops and their strength can be determined by the experimental data. In the literature, there are several studies on LFV...
interactions in different models. Such interactions are studied in a model independent way in [4], in the framework of model III 2HDM [5] and in supersymmetric models [6–12].

Our work is devoted to the study of the $\tau \rightarrow \mu \bar{\nu}_i \nu_i$, $i = e, \mu, \tau$ decay in the model III version of 2HDM. This process exists with the help of internal scalar bosons $h^0$ and $A^0$ to obtain the flavour changing transition $\tau \rightarrow \mu$ and the internal $Z$ boson to get the output $\bar{\nu}_i \nu_i$ (see figure 1). The $BR(\tau \rightarrow \mu \bar{\nu}_i \nu_i)(i = e, \mu, \tau)$ is predicted at the order of magnitude $10^{-6}$–$10^{-4}$ for intermediate values of the Yukawa couplings, which are the free parameters of the model used, and is strongly sensitive to the couplings for $\tau \rightarrow h^0(A^0)\tau$ and $\tau \rightarrow h^0(A^0)\mu$ transitions. This can play an important role in the determination of the upper limits of these couplings, especially that for the $\tau \rightarrow h^0(A^0)\tau$ transition. Note that there have been some experimental studies on this process in the literature [13].

The paper is organized as follows. In section 2, we present the theoretical expression for the decay width of the LFV decay $\tau \rightarrow \mu \bar{\nu}_i \nu_i$, $i = e, \mu, \tau$, in the framework of model III. Section 3 is devoted to discussion and our conclusions.

2. $\tau \rightarrow \mu \bar{\nu}_i \nu_i$ decay in the general two Higgs doublet model

Type III 2HDM permits flavour changing neutral currents (FCNC) at tree level. The Yukawa interaction for the leptonic sector in model III is

$$L_Y = \eta_{ij}^{E} \bar{l}_i L \phi_1 E_{jR} + \xi_{ij}^{E} \bar{l}_i L \phi_2 E_{jR} + h.c.,$$

where $i, j$ are family indices of leptons, $L$ and $R$ denote chiral projections $L(R) = \frac{1}{2}(1 \mp \gamma_5)$, $\phi_i$ for $i = 1, 2$ are the two scalar doublets, $l_{iL}$ and $E_{jR}$ are lepton doublets.
and singlets, respectively. Here $\phi_1$ and $\phi_2$ are chosen as

$$\phi_1 = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \sqrt{Z} \chi^+ \\ i \chi^0 \right] \, , \quad \phi_2 = \frac{1}{\sqrt{2}} \left( \sqrt{2} H^+ \right) \, , \quad$$

(2)

with the vacuum expectation values

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \, , \quad \langle \phi_2 \rangle = 0 \, .$$

(3)

By considering the gauge and $CP$ invariant Higgs potential which spontaneously breaks $SU(2) \times U(1)$ down to $U(1)$ as

$$V(\phi_1, \phi_2) = c_1 (\phi_1^0 \phi_1 - v^2/2)^2 + c_2 (\phi_2^0 \phi_2)^2 + c_3 \left[ (\phi_1^0 \phi_1 - v^2/2) + \phi_2^0 \phi_2 \right]^2$$

$$+ c_4 \left[ (\phi_1^0 \phi_1)(\phi_2^0 \phi_2) - (\phi_1^0 \phi_2)(\phi_2^0 \phi_1) \right] + c_5 [\text{Re}(\phi_1^0 \phi_2)]^2 + c_6 [\text{Im}(\phi_1^0 \phi_2)]^2 + c_7,$$

(4)

with constants $c_i, i = 1, \ldots, 7$, $H_1$ and $H_2$ are obtained as the mass eigenstates $h^0$ and $A^0$ respectively, since no mixing occurs between two $CP$-even neutral bosons $H^0$ and $h^0$ in the tree level. Therefore, it is possible to collect the SM particles in the first doublet and the new particles in the second one. The part which produces FCNC at tree level is

$$L_{Y, FC} = \xi_k^{ij} f_{iL} f_{jR} + h.c..$$

(5)

Here the Yukawa matrices $\xi_k^{ij}$ have complex entries in general. Note that, in the following, we replace $\xi_k^L$ with $\xi_k^R$ where ‘N’ denotes the word ‘neutral’.

Now, we consider the lepton flavour changing process $\tau \to \mu \bar{v} v$ and we expect that the main contribution to this decay comes from the neutral Higgs bosons, namely, $h_0$ and $A_0$ in the loop level, in the leptonic sector of model III (see figure 1). The general effective vertex for the interaction of an off-shell Z-boson with a fermionic current is obtained as

$$\Gamma_\mu^{(\text{REN})}(\tau \to \mu Z^+) = f_1 \gamma_\mu + f_2 \gamma_\mu f_3 + f_3 \gamma_\mu k^\nu + f_4 \gamma_\mu \gamma_5 k^\nu$$

(6)

where $k$ is the momentum transfer, $k^2 = (p - p')^2$, $p$ ($p'$) is the 4-momentum vector of the incoming (outgoing) lepton. Taking into account all the masses of internal ($m_l$) and external leptons ($m_i$), the explicit expressions for the functions $f_1, f_2, f_3$ and $f_4$ are

$$f_1 = \frac{g}{64\pi^2 \cos \theta_W} \int_0^1 \frac{dx}{m_{l_2}^2 - m_{l_1}^2} \left\{ c_V \left( m_{l_1} + m_{l_2} \right) \left( -m_{l_1} \eta_{l_1}^+ + m_{l_2} \left( -1 + x \right) \eta_{l_2}^+ \right) \ln \frac{L_{1,b}^{1,0}}{\mu^2} \right.$$

$$+ \left( m_{l_1} \eta_{l_1}^+ - m_{l_2} \left( -1 + x \right) \eta_{l_2}^+ \right) \ln \frac{L_{1,b}^{1,0}}{\mu^2} + \left( m_{l_1} \eta_{l_2}^+ + m_{l_2} \left( -1 + x \right) \eta_{l_1}^+ \right) \ln \frac{L_{1,b}^{2,0}}{\mu^2}$$

$$- \left( m_{l_1} \eta_{l_1}^+ + m_{l_2} \left( -1 + x \right) \eta_{l_2}^+ \right) \ln \frac{L_{2,b}^{2,0}}{\mu^2} + c_A \left( m_{l_1}^2 - m_{l_2}^2 \right) \right.$$
\[
- (1 - x - y)m_i \left( c_A (m_i - m_l) \eta_l^+ \left( \frac{1}{L_{A_0}} - \frac{1}{L_{A_0}^\prime} \right) + c_V (m_i + m_l) \right)
\times \eta_l^+ \left( \frac{1}{L_{A_0}^\prime} + \frac{1}{L_{A_0}} \right) - \left( c_A \eta_l^+ + c_V \eta_l^V \right) \left( -2 + (k^2 xy + m_i m_l (-1 + x + y)^2) \right)
\times \left( \frac{1}{L_{A_0}^\prime} + \frac{1}{L_{A_0}} \right) - \ln \frac{L_{A_0}^\prime}{\mu^2} \ln \frac{L_{A_0}}{\mu^2} - (m_i + m_l)(1 - x - y)
\times \left( \eta_l^+ (x m_i + y m_l) + m_i \eta_l^+ - \eta_l^+ (x m_i + y m_l) - m_i \eta_l^+ \right)
\times \frac{1}{2 L_{A_0}^\prime} \left( \frac{1}{L_{A_0}^\prime} + \frac{1}{L_{A_0}} \right)
\times \frac{1}{2 L_{A_0}^\prime} \ln \frac{L_{A_0}^\prime}{\mu^2} \ln \frac{L_{A_0}}{\mu^2} \right) \}
\]

\[
f_2 = \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 \left( \frac{1}{m_{l_2}^2 - m_{l_1}^2} \right) \left( c_V (m_{l_2} - m_{l_1}) \eta_{l_1}^- \left( \frac{1}{L_{A_0}^\prime} - \frac{1}{L_{A_0}} \right) + c_V (m_{l_2} + m_{l_1}) \right)
\times \eta_{l_1}^- \left( \frac{1}{L_{A_0}^\prime} + \frac{1}{L_{A_0}} \right) - \left( c_A \eta_{l_1}^- + c_V \eta_{l_1}^V \right) \left( -2 + (k^2 xy + m_{l_2} m_{l_1} (-1 + x + y)^2) \right)
\times \left( \frac{1}{L_{A_0}^\prime} + \frac{1}{L_{A_0}} \right) - \ln \frac{L_{A_0}^\prime}{\mu^2} \ln \frac{L_{A_0}}{\mu^2} - (m_{l_2} + m_{l_1})(1 - x - y)
\times \left( \eta_{l_1}^- (x m_{l_2} + y m_{l_1}) + m_{l_2} \eta_{l_1}^- - \eta_{l_1}^- (x m_{l_2} + y m_{l_1}) - m_{l_2} \eta_{l_1}^- \right)
\times \frac{1}{2 L_{A_0}^\prime} \left( \frac{1}{L_{A_0}^\prime} + \frac{1}{L_{A_0}} \right)
\times \frac{1}{2 L_{A_0}^\prime} \ln \frac{L_{A_0}^\prime}{\mu^2} \ln \frac{L_{A_0}}{\mu^2} \right) \}
\]

\[
f_3 = -i \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 \left( \frac{1}{m_{l_2}^2 - m_{l_1}^2} \right) \left( c_V (m_{l_2} - m_{l_1}) \eta_{l_1}^- \left( \frac{1}{L_{A_0}^\prime} - \frac{1}{L_{A_0}} \right) + c_V (m_{l_2} + m_{l_1}) \right)
\times \eta_{l_1}^- \left( \frac{1}{L_{A_0}^\prime} + \frac{1}{L_{A_0}} \right) - \left( c_A \eta_{l_1}^- + c_V \eta_{l_1}^V \right) \left( -2 + (k^2 xy + m_{l_2} m_{l_1} (-1 + x + y)^2) \right)
\times \left( \frac{1}{L_{A_0}^\prime} + \frac{1}{L_{A_0}} \right) - \ln \frac{L_{A_0}^\prime}{\mu^2} \ln \frac{L_{A_0}}{\mu^2} - (m_{l_2} + m_{l_1})(1 - x - y)
\times \left( \eta_{l_1}^- (x m_{l_2} + y m_{l_1}) + m_{l_2} \eta_{l_1}^- - \eta_{l_1}^- (x m_{l_2} + y m_{l_1}) - m_{l_2} \eta_{l_1}^- \right)
\times \frac{1}{2 L_{A_0}^\prime} \left( \frac{1}{L_{A_0}^\prime} + \frac{1}{L_{A_0}} \right)
\times \frac{1}{2 L_{A_0}^\prime} \ln \frac{L_{A_0}^\prime}{\mu^2} \ln \frac{L_{A_0}}{\mu^2} \right) \}
\]
in the well-known expression
\[ \xi E \]
\[ \text{bosons} \]
In the case of vanishing charged interactions, under the assumption that a CKM type matrix
\[ \nu \rightarrow \tau \]
\[ \text{3. Discussion} \]
where
\[ f_a = -i \frac{g}{64\pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ (1 - x - y) \left( -c_\nu \eta_i^\nu + c_\eta \eta_i^\nu \right) (x m_{\nu i} - y m_{\nu i}) \right\} \]
\[ - m_i (c_\nu (x - y) \eta_i^\nu + c_\nu \eta_i^\nu (x + y)) \left( \frac{1}{L_{\nu \nu}^{\nu \nu}} \right) \]
\[ \times (x m_{\nu i} - y m_{\nu i}) + m_i (c_\nu (x - y) \eta_i^\nu + c_\nu \eta_i^\nu (x + y)) \left( \frac{1}{L_{\nu \nu}^{\nu \nu}} \right) (1 - x - y) \]
\[ \times \left( \frac{\eta_i^\nu}{2} (m_{\nu i} x - m_{\nu j} y) \right) \left( \frac{1}{L_{\nu \nu}^{\nu \nu}} \right) + \left( \frac{1}{L_{\nu \nu}^{\nu \nu}} \right) \]
\[ \left( \frac{1}{L_{\nu \nu}^{\nu \nu}} \right) \left( \frac{1}{L_{\nu \nu}^{\nu \nu}} \right) \left( \frac{1}{L_{\nu \nu}^{\nu \nu}} \right) \left( \frac{1}{L_{\nu \nu}^{\nu \nu}} \right) \left( \frac{1}{L_{\nu \nu}^{\nu \nu}} \right) \]
\[ \text{where} \]
\[ L_{\nu \nu}^{\nu \nu} = m^2_{\nu \nu}(1 - x) + \left(m^2_{\nu i} - m^2_{\nu i}(1 - x)\right)x, \]
\[ L_{\nu \nu}^{\nu \nu} = m^2_{\nu \nu}(1 - x) + m^2_{\nu i}(x + y) - k^2xy, \]
\[ L_{\nu \nu}^{\nu \nu} = m^2_{\nu \nu}(1 - x - y) + \left(m^2_{\nu i} - k^2x\right)y, \]
\[ \eta_i^\nu = \xi_{E,\nu i}^E \xi_{E,\nu i}^E + \xi_{E,\nu i}^E \xi_{E,\nu i}^E, \]
\[ \eta_i^A = \xi_{N,\nu i}^E \xi_{N,\nu i}^E - \xi_{N,\nu i}^E \xi_{N,\nu i}^E, \]
\[ \eta_i^\tau = \xi_{E,\nu i}^E \xi_{E,\nu i}^E + \xi_{E,\nu i}^E \xi_{E,\nu i}^E, \]
\[ \eta_i^\tau = \xi_{E,\nu i}^E \xi_{E,\nu i}^E - \xi_{E,\nu i}^E \xi_{E,\nu i}^E, \]
\[ \text{and} \]
The parameters \( c_\nu \) and \( c_\eta \) are \( c_\nu = -\frac{1}{2} \) and \( c_\nu = \frac{1}{2} - \sin^2 \theta_W \). In equation (9) the flavour changing couplings \( \xi_{\nu i}^E \) represent the effective interaction between the internal lepton \( i \), \( (i = e, \mu, \tau) \) and the outgoing (incoming) \( j = 1 \) \( (j = 2) \) one. Here we take only the \( \tau \) lepton in the internal line and we neglect all the Yukawa couplings except \( \xi_{E,\nu i}^E \) and \( \xi_{E,\nu i}^E \) in the loop contributions (see section 3). Note that the parameter \( \mu \) in equation (7) is the renormalization scale, the functions \( f_1, f_2, f_3, f_4 \) are finite and independent of \( \mu \).

The matrix element \( M \) for the process \( \tau \rightarrow \mu \bar{\nu}_i \nu_i, i = e, \mu, \tau \) is calculated in the framework of the model III by connecting the \( \tau \rightarrow \mu \) transition and the \( \bar{\nu}_i \nu_i \) output with the help of the internal Z boson. Finally, the decay width \( \Gamma \) is obtained in the \( \tau \) lepton rest frame using the well-known expression
\[ d\Gamma = \frac{(2\pi)^4}{2m_\tau} |M|^2 \delta^4 \left( p - \sum_{i=1}^3 p_i \right) \prod_{i=1}^3 \frac{d^3p_i}{(2\pi)^3 2E_i}, \]
where \( p \) \( (p_i, i = 1, 2, 3) \) is the 4-momentum vector of the \( \tau \) lepton (\( \mu \) lepton, incoming \( \nu \), outgoing \( \nu \)).

**3. Discussion**

In the case of vanishing charged interactions, under the assumption that a CKM type matrix in the lepton sector does not exist, LFV interactions arise with the help of the neutral Higgs bosons \( h^0 \) and \( A^0 \) in the framework of model III. In this scenario the Yukawa couplings \( \xi_{E,\nu i}^E, i, j = e, \mu, \tau \) play the main role in the determination of the \( BR \) of the processes under consideration. Since these couplings are free parameters of the theory, they should be restricted by using the present experimental limits of physical quantities, such as \( BR \) of various leptonic
decays and electric dipole moments (EDM), anomalous magnetic moments (AMM) of leptons. In general, these Yukawa couplings are complex and they ensure non-zero lepton EDM.

Now, we briefly discuss the case that the known light neutrinos $\nu_i$ as massive particles where the lepton numbers $L_i$ denote the leptons of $i$th family, are not conserved. If this is so, the lepton sector is an exact analogy to the quark sector and there exists a similar CKM type matrix, the Maki–Nakagawa–Sakata (MNS) matrix $V_{i\nu}$ [14], where its elements are measured in neutrino oscillation experiments. It has been shown that the mixing between the muon neutrino and the heaviest mass eigenstate of the neutrino sector, the $V_{\mu 3}$ element, is nearly maximal [15, 16]. The experiments on solar neutrinos [15, 17] (reactor experiments such as CHOOZ [18]) predicted mixing between the electron neutrino and the second heaviest mass eigenstate of the neutrino sector, the $V_{e2}$ element (the heaviest mass eigenstate of the neutrino sector, the $V_{e3}$ element). Note that the corner element $V_{e3}$ is much smaller than the others. On the other hand, some off-diagonal elements of the MNS matrix, such as $V_{\mu 3}$, are large and the MNS matrix is far from diagonal in contrast to the CKM matrix (see, for example, [19] for more discussion on lepton mixing).

With the inclusion of the MNS matrix, the existence of lepton flavour violating (LFV) processes, the $\tau \rightarrow \mu$ transition in the present work, in the SM, would be possible. However, we expect that the tiny masses of the internal neutrinos make small contribution even with the choice of maximal mixing in the leptonic sector (see, for example, [20] for the discussion of the existence of the MNS matrix and its effects on a special LFV process). In any case, the process $\tau \rightarrow \mu \bar{\nu}_i \nu_i, i = e, \mu, \tau$ should be examined theoretically if the lepton mixing is switched on.

In the framework of model III the $\tau \rightarrow \mu Z^*$ transition can be switched on with the internal neutral Higgs bosons $h^0$ and $A^0$, and internal leptons $e, \mu, \tau$. This brings unknown free parameters $\xi_{N,\mu j}$ and $\xi_{N,\tau j}$, $i, j = e, \mu, \tau$. By assuming that only the internal $\tau$ lepton contribution is considerable, the Yukawa couplings which do not contain index $\tau$ can be neglected. Such a choice respects the statement that the strength of the couplings is related to the masses of leptons denoted by their indices, similar to the Cheng–Sher scenario [21]. Furthermore, we take $\xi_{N,ij}$ symmetric with respect to the indices $i$ and $j$. Finally, we are left with the couplings $\xi_{N,\tau \tau}$ and $\xi_{N,\tau \mu}$, which are complex in general. Note that in the following we use the parametrization:

$$
\xi_{E_{N,ij}} = \sqrt{\frac{4G_F}{\sqrt{2}}} \xi_{E_{N,ij}} \xi_{N,\tau \mu},
$$

and in Table 1 we present the numerical values of some parameters used in the calculations.

The measurement of the $BR$s of $\tau \rightarrow \mu \bar{\nu}_i \nu_i, i = e, \mu, \tau$ decays [13] is based on counting the number of candidate jets and correcting for efficiency and event selection. In addition to this, the backgrounds coming from taus decaying to hadrons or cosmic rays should be

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$m_\tau$</td>
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<tr>
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<tr>
<td>$m_W$</td>
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<td>$s_\tau$</td>
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<td>$G_F$</td>
<td>$1.66 \times 10^{-5}$ (GeV$^{-2}$)</td>
</tr>
<tr>
<td>$\Gamma_Z$</td>
<td>2.49 (GeV)</td>
</tr>
<tr>
<td>$\Gamma_\tau$</td>
<td>$2.27 \times 10^{-12}$ (GeV)</td>
</tr>
</tbody>
</table>
In the process we study, the output contains $\bar{\nu}_i\nu_i$, $i = e, \mu, \tau$, and the extraction of this output from the most probable one $\bar{\nu}_\mu\nu_\mu$ ($BR \sim 17.37 \pm 0.09\%$ [22]), which exist in the SM theoretically, is difficult from the experimental point of view. In this work, we studied the $BR$ of the process $\tau \rightarrow \mu \bar{\nu}_i\nu_i$, $i = e, \mu, \tau$ and we used the numerical value $\xi_{N,\tau\mu}^E$ in the interval $5 \text{ GeV} < |\xi_{N,\tau\mu}^E| < 25 \text{ GeV}$. Here, the upper limit for the coupling $|\xi_{N,\tau\mu}^E|$ has been estimated in [23] as $\sim 30 \text{ GeV}$. In this work it is assumed that the new physics effects are of the order of the experimental uncertainty of muon AMM, namely $10^{-9}$, by respecting the new experimental world average announced at BNL [24].

$$a_\mu = 11659203 \times 10^{-10},$$

which has about half the uncertainty of previous measurements. Here we have not used any restriction for the coupling $\bar{\xi}_{N,\tau\tau}^E$ except that we choose its numerical value larger compared to $\xi_{N,\tau\mu}^E$. In addition to this, we expect the upper limit of $\xi_{N,\tau\tau}^E$ by taking the upper limit of $\xi_{N,\tau\mu}^E$ and the $BR$ of the process as $10^{-5} - 10^{-6}$.

In figure 2, we present the $\xi_{N,\tau\tau}^E$ dependence of the $BR$ for real couplings. Here the solid (dashed, small dashed, dotted, dash-dotted) line represents the case for $\xi_{N,\tau\mu}^E = 5 \text{ GeV}$ (10, 15, 20, 25 GeV). This figure shows that the $BR$ is enhanced with increasing values of both couplings and it reaches values of the order of magnitude $10^{-4}$. Figure 3 shows the possible values of $\xi_{N,\tau\tau}^E$ and the ratio $\xi_{N,\tau\tau}^E/\xi_{N,\tau\mu}^E$ for fixed values of the $BR, BR = 10^{-4}$ (solid line) and $BR = 10^{-6}$ (dashed line). For $\bar{\xi}_{N,\tau\tau}^E = 0.1, BR = 10^{-4}$ is obtained if the coupling $\bar{\xi}_{N,\tau\tau}^E \sim 150 \text{ GeV}$ and the $BR = 10^{-6}$ is obtained if the coupling $\bar{\xi}_{N,\tau\tau}^E \sim 50 \text{ GeV}$. The possible experimental search for the process $\tau \rightarrow \mu \bar{\nu}_i\nu_i$, $i = e, \mu, \tau$ would ensure a strong clue in the prediction of the upper limit of the coupling $\bar{\xi}_{N,\tau\tau}^E$.

Figure 4 represents the $h^0$ mass $m_{h^0}$ dependence of the $BR$ for the fixed values of $\xi_{N,\tau\mu}^E$ and $\xi_{N,\tau\tau}^E = 10 \text{ GeV}, \xi_{N,\tau\tau}^E = 100 \text{ GeV}$. This figure shows that the $BR$ is sensitive to $m_{h^0}$ and it decreases with increasing values of $m_{h^0}$.

Now, we take the coupling $\bar{\xi}_{N,\tau\tau}^E$ complex

$$\bar{\xi}_{N,\tau\tau}^E = |\bar{\xi}_{N,\tau\tau}^E| e^{i\theta_{h^0}}.$$
and present the $\sin \theta_{\tau\tau}$ dependence of the BR for $\xi_{N,\tau\tau}^E = 100$ GeV for four different values of $\xi_{N,\tau\mu}^E$, namely $\xi_{N,\tau\mu}^E = 5, 10, 15, 20$ GeV (solid, dashed, small dashed, dotted lines) in figure 5. From this figure it can be shown that the BR is not sensitive to the complexity of the coupling $\xi_{N,\tau\tau}^E$.

At this stage we would like to summarize our results:

- We predict the BR at the order of magnitude $10^{-6} - 10^{-4}$ for the range of couplings, $\xi_{N,\tau\tau}^E \sim 30 - 100$ GeV and $\xi_{N,\tau\mu}^E \sim 10 - 25$ GeV. We predict the upper limit of the coupling for the $\tau - h^0(A^0) - \tau$ transition as $\sim 0.3$ in the case that the BR is $\sim 10^{-6}$. With the possible experimental measurement of the process $\tau \rightarrow \mu \nu_i \nu_i, i = e, \mu, \tau$, a strong clue in the prediction of the upper limit of the coupling $\xi_{N,\tau\tau}^E$ would be obtained. Note that the upper limit of the coupling $\xi_{N,\tau\mu}^E$ could be predicted previously (see, for example, [23]). This analysis also ensures a hint for the physics beyond the SM.
We observe that the $BR$ is sensitive to the neutral Higgs masses $m_{h^0}$ and $m_{A^0}$.

We observe that the $BR$ is not sensitive to the possible complexity of the Yukawa couplings.

Therefore, the future theoretical and experimental investigations of the process $\tau \rightarrow \mu \bar{\nu}_i \nu_i$ would be informative in the determination of the physics beyond the SM and the free parameters existing in this model.

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