Scalar quarkonia at finite temperature

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Masses and decay constants of the scalar quarkonia, \( \chi_{Q0}(Q = b, c) \) with quantum numbers \( I^G(J^{PC}) = 0^+(0^{++}) \), are calculated in the framework of the QCD sum rules approach both in vacuum and finite temperature. The masses and decay constants remain unchanged up to \( T \approx 100 \text{ MeV} \) but they start to diminish with increasing the temperature after this point. Near the critical or deconfinement temperature, the decay constants reach approximately to 25% of their values in vacuum, while the masses are decreased about 6% and 23% for bottom and charm cases, respectively. The results at zero temperature are consistent with the existing experimental values and predictions of the other nonperturbative approaches. Our predictions on the decay constants in vacuum, as well as the behavior of the masses and decay constants with respect to the temperature, can be checked in the future experiments.

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I. INTRODUCTION

Understanding the internal structure of scalar mesons has been a prominent topic in the last 30–40 years. Although the scalar mesons have been investigated for several decades, many properties of them are not clear yet, and identifying the scalar mesons is difficult experimentally. Hence, the theoretical works can play a crucial role in this respect. In particle physics, the quarkonia refers to flavorless mesons containing a heavy \( b \) (\( c \)) quark and its own antiquark, i.e., \( b\bar{b} \) (bottomonium) and \( c\bar{c} \) (charmonium). These approximately nonrelativistic systems are the best candidates to investigate the hadronic dynamics and to study the perturbative and nonperturbative aspects of QCD. It was believed that the quarkonia can help us to extract the nature of quark-antiquark interaction at the hadronic scale and play the same role in probing the QCD as the hydrogen atom play in the atomic physics [1].

A large number of the beauty and charmed systems have been experimentally observed in the last few decades (see, for instance, [2–4]) and the theoretical calculations on the properties of these systems have been made mainly using potential model, where the quarkonia is described by a static potential, \( V = -\frac{3}{2} \frac{a_0}{r} + k r \) and its extensions like the Coulomb gauge model [5–9]. The first term in the potential is related to one gluon exchange, and the second term is called the confinement potential. The recent CLEO measurements on the two-photon decay rates of the even-parity, \( P \)-wave scalar \( 0^{++}, \chi_{b(c)0} \) and tensor \( 2^{++}, \chi_{b(c)2} \) states ([10,11], and references therein) were the motivation to investigate the properties of the quarkonia and their radiative decays from the quark-antiquark interaction point of view (see, for example, [12,13]).

In [14], which is a recent study on extraction of ground-state decay constant from both sum rules and potential models, it is stated that results obtained at each step of the extraction procedure both in QCD and in potential models follow the same pattern; hence all of our findings concerning the extraction of bound-state parameters from correlation functions obtained in potential model can apply also to QCD. It is also proven that in the QCD sum rules approach, by tuning the continuum threshold which is related to the energy of the first exited state specially with a Borel parameter-dependent threshold, we can get a more reliable and accurate determination of bound-state characteristics comparing the potential models. The QCD sum rules approach as a nonperturbative approach is one of the most powerful and applicable tools to studying the spectroscopy of hadrons and can play a crucial role in the investigation of the properties of the hadrons [15–18]. It has been used to calculate the masses and decay constants of mesons [19–26]. This approach was extended to contain the properties of the hadrons at finite temperature called thermal QCD sum rules [27–29] supposing that the operator product expansion (OPE) and the quark-hadron duality assumption remain valid; however, the quark-quark, quark-gluon, and gluon-gluon condensates are altered by their thermal versions. The main aspiration of this addendum was to explain the results obtained from the heavy ion collision experiments. It is presently believed that the hot and dense medium where the hadrons are formed modifies masses and decay widths of hadrons. It is shown that heavy mesons like \( J/\psi \) and also radial and orbital \( c\bar{c} \) excitations have different behaviors when the temperature of the medium changes (see [30], and references therein). In [31], scalar mesons and scalar glueballs are investigated in holographic QCD at finite temperature. A flood of papers have also been dedicated generally to the

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determination of the condensates, mass and decay constant of mesons, and some properties of the nucleons at finite temperature [32–45].

In the present work, we calculate the mass and decay constant of the heavy scalar \( \chi_{SS} \) mesons with quantum numbers \( I^G(J^{PC}) = 0^-(0^{++}) \) using the thermal QCD sum rules approach. Here, we assume that with replacing the vacuum condensates and also the continuum threshold by their thermal version, the sum rules for the observables (masses and decay constants) remain valid. In calculations, we take into account the additional operators in the Wilson expansion at finite temperature [46] and modify spectral density in the QCD side. These operators are due to the breakdown of Lorentz invariance at finite temperature by density in the QCD side. These operators are due to the residual O(3) invariance brings these extra operators with the same mass dimension as the vacuum condensates. We also consider the interaction of the currents with the vacuum can be written in terms of the decay constant, \( f_S \) by the following manner:

\[
\langle 0 | J(0) | S \rangle = f_S m_S.
\]

Note that Eqs. (3) and (4) are valid also at finite temperature, hence, the final representation for the physical side can be written in terms of the temperature dependent mass and decay constant as

\[
\Pi(q, T) = \frac{f_S^2(T)m_S^2(T)}{m_S^2(T) - q^2} + \cdots.
\]

In the QCD side, the correlation function is calculated in deep Euclidean region, \( q^2 \ll -\Lambda_{QCD}^2 \) via OPE where the short or perturbative and long distance or nonperturbative effects are separated, i.e.,

\[
\Pi_{QCD}(q, T) = \Pi_{pert}(q, T) + \Pi_{nonpert}(q, T).
\]

The short distance contribution [bare loop diagram in Figure 1(a)] is calculated using the perturbation theory; whereas, the long distance contributions [diagrams shown in Figure 1(b)] are represented in terms of the thermal expectation values of some operators. To proceed, we write the perturbative part in terms of a dispersion integral,

\[
\Pi_{QCD}(q, T) = \int \frac{d\rho(s, T)}{s - q^2} + \Pi_{nonpert},
\]

where, \( \rho(s, T) \) is called the spectral density at finite temperature. The thermal spectral density at fixed \( |q| \) can be expressed as
\[
\rho(q,T) = \frac{1}{\tau} \text{Im} \Pi^{pert}(q,T) \tanh \left( \frac{\beta q_0}{2} \right). \tag{8}
\]

To proceed, we need to know the fermion propagator at finite temperature. The thermal fermion propagator at real time is given as
\[
S(k) = (\gamma_\mu k^\mu + m_Q) \\
\times \left( \frac{i}{k^2 - m_Q^2 + i\epsilon} - 2\pi n(k_0) \delta(k^2 - m_Q^2) \right). \tag{9}
\]

and here, \( n(x) \) is Fermi distribution function,
\[
n(x) = [\exp(\beta x) + 1]^{-1}. \tag{10}
\]

Using the above propagator, we find the following expression for the imaginary part of the correlation function at \( |q| = 0 \) limit:
\[
\text{Im} \Pi(q_0,T) = N_c \int \frac{dk}{8\pi^2} \frac{1}{\omega^2} (q_0 \omega - 2m_Q^2) \\
\times (1 - 2n(\omega) + 2n^2(\omega)) \delta(q_0 - 2\omega), \tag{11}
\]

where, \( \omega = \sqrt{m_Q^2 + k^2} \). After some straightforward calculations, the thermal spectral density is obtained as
\[
\rho(s) = \frac{3s}{8\pi^2} \left( 1 - \frac{4m_Q^2}{s} \right)^{3/2} \left( 1 - 2n(\sqrt{s}) \right). \tag{12}
\]

In the nonperturbative part, the main contribution comes from the two gluon condensates since the heavy quark condensates are suppressed by inverse powers of the heavy quark mass and can be safely removed. The gluon condensate diagrams are represented in part (b) of Fig. 1. In order to calculate nonperturbative contributions, we use Fock-Schwinger gauge, \( x^\mu A_\mu^g (x) = 0 \). In momentum space, the vacuum gluon field is expressed as
\[
A_\mu^g (k') = -\frac{i}{2} (2\pi)^4 G_{\rho \mu}^g (0) \frac{\partial}{\partial k^\rho} g^{(4)}(k'), \tag{13}
\]

and in calculations, we use the quark-gluon-quark vertex as
\[
\Gamma_{ij\mu} = ig \gamma_\mu \left( \frac{\Lambda^4}{2} \right)_{ij}, \tag{14}
\]

where \( k' \) is the gluon momentum.

After straightforward calculations, the nonperturbative part in momentum space is obtained as
\[
\Pi^{pert} = \int_0^1 dx \frac{x^2}{288\pi (m_Q^2 + q^2 x(-1 + x))^2} \left\{ 3(\alpha_s G^2) [40m_Q^6 x^2 - 9q^6 x^2 (-1 + x)^4 (-1 - 2x + 2x^2) \\
- 12m_Q^2 q^4 x(-1 + x)^2 (1 + 4x - 12x^2 + 6x^3) + m_Q^4 q^2 (-15 + 156x - 441x^2 + 434x^3 - 134x^4)] \\
- \alpha_s (u^a \Theta_{a \beta}^g u^\beta) [4q^4 (-1 + x)(q^4 (-1 + x)^2 x^2 (9 + 11x - 14x^2 + 12x^3) + m_Q^4 (-15 + 153x - 246x^2 + 176x^3) + 4m_Q^2 q^2 (-3 - 8x + 28x^2 - 34x^3 + 17x^4)) - 16(-1 + x)(q,u)^2 (q^4 x^2 (-1 + x)^2 (9 + 11x - 14x^2 + 12x^3) \\
+ m_Q^4 (-15 + 135x - 246x^2 + 176x^3) + 4m_Q^2 q^2 x(-3 - 8x + 28x^2 - 34x^3 + 17x^4))] \right\}, \tag{15}
\]

where the 4-vector \( u^\mu \) is the velocity of the heat bath and it is introduced to restore Lorentz invariance formally in the thermal field theory. In the rest frame of the heat bath, \( u^\mu = (1, 0, 0, 0) \) and \( u^2 = 1 \). In deriving the above expression, we have used the following relation considering the Lorentz covariance [37]:
\[
(\text{Tr}^c G_{a \beta} G_{\lambda \sigma}) = \left( g_{a \lambda} g_{\beta \sigma} - g_{a \sigma} g_{\beta \lambda} \right) A - (u_a u_\lambda g_{\beta \sigma} \\
- u_\mu u_\sigma g_{\beta \lambda} - u_\mu u_{\beta} g_{\lambda \sigma} + u_{\mu} u_{\sigma} g_{\lambda \alpha}) B. \tag{16}
\]

Contracting indices on both sides, we obtain
\[
A = \frac{1}{2\pi}(G^a_{a \beta} G^{a \alpha \beta}) + \frac{1}{6}(u^a \Theta_{a \beta}^g u^\beta). \tag{17}
\]

\[
B = \frac{1}{3}(u^a \Theta_{a \beta}^g u^\beta). \tag{18}
\]

where \( \Theta_{a \beta}^g \) is the traceless, gluonic part of the stress-tensor of the QCD, and it is defined as
\[
\Theta_{a \beta}^g = -G_{a \lambda}^g G_{\beta}^{\lambda} + \frac{1}{\delta} g_{a \beta} G_{\sigma}^{\sigma} G^{\lambda \sigma}. \tag{19}
\]

Matching the phenomenological and QCD sides of the correlation function, the sum rules for the mass and decay constant of scalar meson are obtained. To suppress the contribution of the higher states and continuum, Borel transformation over \( q^2 \), as well as continuum subtraction, are performed. As a result of the above procedure, we obtain the following sum rule for the decay constant:

\[
\begin{align*}
\end{align*}
\]

\[
\begin{align*}
\end{align*}
\]
described by the expression
\[ T_s \text{ where,} \]
\[ s(0) = \frac{d}{d(1/M^2)} \tilde{B} \Pi_{\text{nonpert}}(M^2, T), \tag{22} \]
and \( \tilde{B} \Pi_{\text{nonpert}}(M^2, T) \) shows contribution of the gluon condensates in Borel transformed scheme and is given by

\[
\tilde{B} \Pi_{\text{nonpert}} = \int_0^1 dx e^{m_0^2/M^2 x(x-1)} \frac{1}{96M^6 \pi(x-1)^4 x^5} \left\{ (\alpha G^2)(m_0^2(1-2x)^2(-3+5x) + 9M^6(-1+x)^4x(-1-2x+2x^2) - 3m_0^2M^4(-1+x)^2x^2(-5+7x-12x^2+6x^3) + 2m_0^4M^2x(3-21x+48x^2-41x^3+11x^4)) - 4\alpha_1(\Theta^s)(m_0^2(1-2x)^2(-3+5x) + M^6(-1+x)^4x(9+11x-14x^2+12x^3) + 2m_0^4M^2x(3-23x+58x^2-57x^3+19x^4) - m_0^2M^4(-1+x)^2x^2(-15+11x-26x^2+32x^3)) \right\}. \tag{23} \]

We use the gluonic part of energy density both obtained from lattice QCD \[38] and chiral perturbation theory \[39].
In the rest frame of the heat bath, the results obtained in \[38] at lattice QCD are reproduced well by the following fit parametrization for the thermal average of total energy density \( \langle \Theta \rangle \):

\[
\langle \Theta \rangle = \langle \Theta^s \rangle + 3p, \tag{25} \]
where, \( p \) is pressure and \( \langle \Theta^s \rangle \) is trace of the total energy momentum tensor. They are given as

\[
\langle \Theta^s \rangle = s(0) \left[ 1 - \left( \frac{T_c}{T_c} \right)^4 \right] + 4m_0^4 \frac{T_c^8}{T^8} \left( \frac{T}{T_c} \right)^8, \tag{27} \]

\[
\langle G^2 \rangle = \frac{\langle 0 | G^2 | 0 \rangle}{\exp(12(\frac{1}{T^2} - 1.05)) + 1}. \tag{28} \]

where \( T_c = 1.1 \times T_c = 0.176 \text{ GeV} \), and \( s(0) = \langle 0 | G^2 | 0 \rangle \) are the continuum threshold and the gluon condensate in vacuum, respectively. These parametrizations are valid only in the interval \( 0 \leq T \leq 170 \text{ MeV} \). Here, we should stress that the continuum threshold presented above is equal to the continuum threshold in vacuum at \( T = 0 \), but it starts to diminish increasing the temperature such that at \( T = T_c^* \) it reaches the perturbative QCD threshold, \( 4m_0^2 \). This parametrization belongs to heavy-heavy system and differ considerably with the case of light-light and heavy-light quark systems, where the continuum threshold is related to the thermal light quark condensate (for details, see [50]).

We also use temperature dependent strong coupling constant \[51,52] as

\[
g^{-2}(T) = \frac{11}{8\pi} \ln \left( \frac{2\pi T}{\lambda_{ MS}} \right) + \frac{51}{88\pi^2} \ln \left[ 2 \ln \left( \frac{2\pi T}{\lambda_{ MS}} \right) \right]. \tag{29} \]

where, \( \lambda_{ MS} \approx T_c/1.14 \) and in numerical calculations, instead of the \( \alpha_s \) in front of \( \langle \Theta^s \rangle \) in Eq. (23) the \( \tilde{\alpha}(T) = 2.096\alpha_{\text{pert}}(T) \) has been used, where \( \alpha_{\text{pert}}(T) = \frac{\varepsilon^2(T)}{4\pi} \) (for details, see [51]).

III. NUMERICAL ANALYSIS

The present section is devoted to the numerical analysis of the sum rules for the mass and decay constant of the heavy scalar mesons. In further analysis, we use \( m_\pi = (1.3 \pm 0.05) \text{ GeV} \), \( m_\eta = (4.7 \pm 0.1) \text{ GeV} \) and \( \langle 0 | \frac{1}{n} \alpha_s G^2 | 0 \rangle = (0.012 \pm 0.004) \text{ GeV}^4 \). The sum rules for
the mass and decay constant also contain two auxiliary parameters, continuum threshold $s_0$, and Borel mass parameter $M^2$. The standard criteria in QCD sum rules is that the physical quantities should be independent of the auxiliary parameters. However, the continuum threshold $s_0$ is not completely arbitrary but is related to the energy of the first exited state with the same quantum numbers and can depend on the Borel mass parameter [53]. Therefore, the standard criteria does not render realistic errors, and in fact the existing error should be large. Hence, we will add also the systematic errors to the numerical results. We choose the values $s_0 = (110 \pm 4)$ GeV$^2$ and $s_0 = (18 \pm 2)$ GeV$^2$ for the continuum threshold at $\chi_{b0}$ and $\chi_{c0}$ channels, respectively. The working region for the Borel mass parameter is determined requiring that not only the higher state and continuum contributions are suppressed, but also the contribution of the highest order operator should be small, i.e., the sum rules for the mass and decay constant should converge. As a result of the above procedure, the working region for the Borel parameter is found to be $8$ GeV$^2 \leq M^2 \leq 20$ GeV$^2$ for $\chi_{c0}$ and $15$ GeV$^2 \leq M^2 \leq 30$ GeV$^2$ for $\chi_{b0}$ mesons. The dependences of the masses and decay constants at $T = 0$ on Borel mass parameter are shown in Figs. 2–5. These figures depict that the observables depend very weakly on the Borel mass parameter, $M^2$ in the working regions.
The dependence of the mass and decay constant of the \( \chi_{b0} \) and \( \chi_{c0} \) mesons on temperature are presented in Figs. 6–9. In these figures, we show the results obtained using both lattice QCD and chiral perturbation limit values for the gluonic part of energy density. These figures depict that the results depend very weakly on the values of the gluonic part of energy density, i.e., both values obtained from lattice and chiral limit have approximately the same predictions in the interval, \( 0.1 \text{ GeV} \leq T \leq 0.17 \text{ GeV} \) at which the lattice results are valid. These figures also show that the masses and decay constants do not change up to \( T \approx 100 \text{ MeV} \), but they start to diminish with increasing the temperature after this point. Near the critical or deconfinement temperature, the decay constants reach approximately to 25% of their values in vacuum, while the masses are decreased about 6% and 23% for bottom and charm cases, respectively. From these figures, we deduce the results on the decay constant and mass in vacuum as presented in Tables I and II. The quoted errors in these Tables are due to the errors in variation of the continuum threshold, Borel mass parameter, and errors in other input parameters as well as the systematic uncertainties.

**Figure 6** (color online). The dependence of the decay constant of scalar \( \chi_{b0} \) meson on temperature obtained using both lattice QCD and chiral perturbation limit values for the gluonic part of energy density. The values \( s_0 = 110 \text{ GeV}^2 \), and \( M^2 = 17 \text{ GeV}^2 \) have been used for the continuum threshold and Borel mass parameter in vacuum, respectively.

**Figure 8** (color online). The dependence of the decay constant of scalar \( \chi_{c0} \) meson on temperature obtained using both lattice QCD and chiral perturbation limit values for the gluonic part of energy density. The values \( s_0 = 18 \text{ GeV}^2 \), and \( M^2 = 9 \text{ GeV}^2 \) have been used for the continuum threshold and Borel mass parameter in vacuum, respectively.

**Figure 7** (color online). The same as Fig. 6 but for mass of the scalar \( \chi_{b0} \) meson.

**Figure 9** (color online). The same as Fig. 8 but for mass of the scalar \( \chi_{c0} \) meson.
TABLE I. Values of the leptonic decay constants of the heavy scalar, $\chi_b0$ and $\chi_c0$ mesons in vacuum. These results have been obtained using the values $s_0 = 110$ GeV$^2$ and $M^2 = 17$ GeV$^2$ for $\chi_b0$, and $s_0 = 18$ GeV$^2$ and $M^2 = 9$ GeV$^2$ for $\chi_c0$ mesons.

<table>
<thead>
<tr>
<th></th>
<th>$f_{\chi_b0}$ (MeV)</th>
<th>$f_{\chi_c0}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present work</td>
<td>175 ± 55</td>
<td>343 ± 112</td>
</tr>
<tr>
<td>QCD sum rules</td>
<td>⋅⋅⋅</td>
<td>359</td>
</tr>
<tr>
<td>Cornell potential model</td>
<td>⋅⋅⋅</td>
<td>338</td>
</tr>
<tr>
<td>QCD sum rules</td>
<td>⋅⋅⋅</td>
<td>510 ± 40</td>
</tr>
</tbody>
</table>

we also compare our predictions on the masses of the heavy scalar mesons with the existing experimental data which are consistent. Our results for the leptonic decay constants, as well as their behavior with respect to the temperature, can be verified in the future experiments.

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