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The investigation of Fermi excitations in a quark–gluon plasma in the lightcone gauge

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Abstract. Fermi excitations in a quark–gluon plasma are investigated in the lightcone gauge. \( \Sigma(p) \) quark self-energy in the one-loop approximation is calculated using the Leibbrandt–Mandelstam prescription. The quark excitation spectrum is found in the high-temperature limit and the dispersion equation is shown to be gauge invariant.

1. Introduction

Collective excitations in a quark–gluon plasma have been widely discussed in the literature. The first discussions of plasma oscillations in quantum chromodynamics appeared in [1–3]. In later studies of collective excitations, the hard thermal loop approximation [4, 5] and the kinetic theory of hard thermal loops [6] have played the most important role.

The aim of this paper is to investigate Fermi excitations in a hot quark–gluon plasma in the lightcone gauge using the standard temperature Green function method. It is well known that the lightcone gauge (in general, \( n\mu A^\mu_\nu(x) = 0 \) non-covariant gauges) has some attractive features. However, the large residual symmetry that is present in this gauge, manifests itself in the appearance of \( 1/nk \) poles in the gluon propagator and various attempts have been made to define these singularities [7–10]. The investigation of quark excitations in the lightcone gauge is very important for two reasons: firstly, to verify the gauge independence of the quark excitation spectrum that was calculated in the covariant gauge, and secondly, to examine the different prescriptions that are used in the lightcone gauge in thermal field theories.

2. The quark excitation spectrum

Collective excitations in a quark–gluon plasma are defined as poles of propagators: the real part of the pole gives the dispersion law, while the imaginary part gives the damping rate. Hence, the quark excitation spectrum is described by the equation

\[
G^{-1}\psi = \left( G^{-1}_0 + \Sigma^{\text{ret}} \right) \psi = 0
\]

where \( G(p) \) and \( G_0(p) \) are exact and bare quark propagators, respectively, and \( \Sigma(p) \) is the quark self-energy. All the functions in (1) must satisfy the retarded (or advanced) boundary...
The quark self-energy in the one-loop approximation in the lightcone gauge has the form
\[ \Sigma(p) = \frac{4g^2}{3\beta} \sum_{k,k_1} \int \frac{d^3k}{(2\pi)^3} \left( \delta_{\mu\nu} - \frac{n_\mu q_\nu + n_\nu q_\mu}{n \cdot q} \right) i\gamma_\nu \gamma_\mu q^2 k^\nu \]

(2)

where \( q = p - k, k_1 = (2n + 1)\pi T, q_4 = 2n\pi T \), the metric chosen is Euclidean and we assume that the quark masses are negligibly small. For explicit calculations, it is convenient to transform \( \Sigma(p) \) into the following form:
\[ \Sigma(p) = \Sigma_1(p) + \Sigma_2(p) + \Sigma_3(p) \]

(3)

where
\[ \Sigma_1(p) = -\frac{8ig^2}{3\beta} \sum_{q_4} \int \frac{d^3q}{(2\pi)^3} \frac{\langle n \rangle (\gamma \cdot q) + \langle \gamma n \rangle (p \cdot q)}{q^2 (p + q)^2 (qn)} \]
\[ \Sigma_2(p) = -\frac{8ig^2}{3\beta} \sum_{q_4} \int \frac{d^3q}{(2\pi)^3} \frac{(\gamma n)}{q^2 (p + q)^2 (qn)} \]
\[ \Sigma_3(p) = -\frac{8ig^2}{3\beta} \sum_{q_4} \int \frac{d^3q}{(2\pi)^3} \frac{(\gamma q)}{q^2 (p + q)^2} \]

The quark self-energy can always be expressed as \( \Sigma(p) = \Sigma_{\text{vac}}(p) + \Sigma_{\text{mat}}(p) \), where \( \Sigma_{\text{vac}}(p) = \lim_{T \to 0} \Sigma(p) \) is the vacuum self-energy and \( \Sigma_{\text{mat}}(p) \) is the remainder due to the presence of matter. In order to calculate \( \Sigma_1(p) \) and \( \Sigma_2(p) \) we use the Leibbrandt–Mandelstam prescription \([7, 8]\), which has the form
\[ \frac{1}{qn} = -\lim_{\epsilon \to 0} \frac{\vec{q} \cdot \vec{n} + i\epsilon q_4}{(\vec{q} \cdot \vec{n})^2 + q_4^2 + \epsilon^2} \]

(4)

where we assume that \( |\vec{n}| = 1 \). After applying this prescription, the summation over the frequencies \( q_4 \) is performed using the standard methods. After some algebraic transformations, \( \Sigma_1(p), \Sigma_2(p) \) and \( \Sigma_3(p) \) can be written as
\[ \Sigma_1(p) = -\frac{8ig^2}{3} \int \frac{d^3\vec{q}}{(2\pi)^3} \left[ \langle pn \rangle (\vec{\gamma} \cdot \vec{q}) + \langle \gamma n \rangle (\vec{p} \cdot \vec{q}) \right] \frac{1}{q^2 - p^2} \left[ I(\rho, |\vec{q}|) - I(\rho = |\vec{q}|, |\vec{q}|) \right] \]
\[ + \frac{8ig^2}{3} \int \frac{d^3\vec{q}}{(2\pi)^3} \left[ \langle pn \rangle \gamma_4 + \langle \gamma n \rangle p_4 \right] \frac{1}{q^2 - p^2} \left[ H(\rho, |\vec{q}|) - H(\rho = |\vec{q}|, |\vec{q}|) \right] \]

(5)

\[ \Sigma_2(p) = -\frac{8ig^2}{3} (\gamma n) \int \frac{d^3\vec{q}}{(2\pi)^3} I(\rho, |\vec{q}|) \]

(6)

\[ \Sigma_3(p) = -\frac{8ig^2}{3} \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{n_B(q) \eta \gamma_4 \cdot \vec{q} - 2p_3 q_4 |\vec{q}|^2}{\eta^2 + 4p_3^2 |\vec{q}|^2} \]
\[ + \frac{8ig^2}{3} \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{n_F(q) \eta \gamma_4 (\vec{p} \cdot \vec{q}) + p_4 (\eta + 2|\vec{q}|^2) \gamma_4}{\eta^2 + 4p_3^2 |\vec{q}|^2} \]

(7)
where

\[ I(\rho, |\vec{q}|) = \frac{1}{2\rho \lambda (\alpha^2 + 4p_4^2\rho^2)} \left[ \lambda (\alpha(\vec{q} \cdot \vec{n}) + 2ip_4\rho^2) n_B(\rho) + (\vec{q} \cdot \vec{n}) (4p_4^2 - \alpha) \right. \]
\[ + i(\alpha p_4 - 2p_4^3 - 2p_4\lambda^2) n_F(\lambda) \] \]
\[ H(\rho, |\vec{q}|) = \frac{1}{2\lambda} \frac{1}{\alpha^2 + 4p_4^2\rho^2} \left[ (2p_4\rho(\vec{q} \cdot \vec{n}) - i\rho\alpha)n_B(\rho) + (\vec{q} \cdot \vec{n})(-2p_4\lambda^2 + 2p_4^3 + \alpha p_4) n_F(\lambda) \right. \]
\[ + (2ip_4(3\lambda^2 p_4 - p_4^3) + (4p_4^2 - \alpha)(p_4^2 - \lambda^2)) n_F(\lambda) \] \]
\[ \eta = p^2 + p_4^2 + 2\vec{p}\vec{q} \quad \lambda = |\vec{p} + \vec{q}| \]
\[ \rho^2 = (\vec{q} \cdot \vec{n})^2 + \varepsilon^2 \quad \alpha = \lambda^2 + p_4^2 - \rho^2. \]

Here \( n_B(\rho) = \left[ \exp(\rho/T) - 1 \right]^{-1} \) and \( n_F(\lambda) = \left[ \exp(\lambda/T) + 1 \right]^{-1} \) are the Bose and Fermi distributions, respectively.

An analysis of equations (5)–(7) in the high-temperature limit shows that \( \Sigma_1(p) \) is proportional to \( T^2 \) and that \( \Sigma_1(p) \) and \( \Sigma_2(p) \) are proportional to \( T \). Therefore, the leading term of \( \Sigma(p) \) arises only from \( \Sigma_1(p) \), which does not include \( 1/nk \) poles. After some transformations and integrations, the high-temperature limit of \( \Sigma_{\text{mat}}(p) \) is given by

\[ \Sigma_{\text{mat}}(p) = \frac{g^2 T^2}{6} \left[ -\frac{i\vec{p}\cdot\vec{q}}{p^2} \left( 1 - p_4^2 \int_0^1 \frac{dz}{p^2 z^2 + p_4^2} \right) - ip_4 \int_0^1 \frac{dz}{p^2 z^2 + p_4^2} \right]. \] (8)

This result coincides with the expression obtained in the covariant gauge. According to equation (1), the quark excitation spectrum in the high-temperature limit is defined by the equation

\[ (p_\mu + \Sigma_\mu)^2 = 0 \] (9)

where \( \Sigma_\mu = \frac{i}{2} \delta_{\mu\nu} \text{Tr} [\gamma_\nu, \Sigma(p)] \). After the standard analytic continuation \( p_4 \rightarrow i(\omega + i\varepsilon) \) we obtain the dispersion equation in the following form:

\[ \left[ \omega - \frac{\omega_0}{\alpha} F \left( \frac{\omega}{|\vec{p}|} \right) \right]^2 = \left( \frac{|\vec{p}| + \omega_0^2}{|\vec{p}|} \left[ 1 - F \left( \frac{\omega}{|\vec{p}|} \right) \right] \right)^2 \] (10)

\[ \omega_0^2 = \frac{g^2 T^2}{6} F(x) = \frac{x}{2} \left[ \log \left| \frac{x + 1}{x - 1} \right| - i\pi \theta (1 - |x|) \right] \]

which is precisely the same equation as that obtained previously in the covariant gauge [2]. Therefore, the quark excitation spectrum obtained in the high-temperature limit is gauge independent, which agrees with the work in [11]. It is also essential to investigate collective excitations and damping rates in other non-covariant gauges. We consider these studies to be of great importance for an understanding of many problems in thermal quantum chromodynamics.

References