The Mass and Leptonic Decay Constant of $D_{s0}(2317)$ Meson in the Framework of Thermal QCD Sum Rules

Elşen Veli Veliev and Gülşah Kaya

Physics Department, Kocaeli University, Umutepe Yerleşkesi
41380, İzmit-TURKEY

Abstract. In the present work, we assume $D_{s0}(2317)$ meson as the $c\bar{s}$ state and study its parameters at finite temperature. Taking into account perturbative two-loop order $\alpha_s$ corrections to the correlation function and nonperturbative corrections up to the dimension six condensates we investigated the temperature dependences of mass and leptonic decay constant using thermal QCD sum rules method.

Keywords: Thermal QCD Sum Rules, Heavy-light Mesons, Leptonic Decay Constant

PACS: 11.10.Wx, 12.40.Yx, 12.38.Lg, 13.20.Fc

INTRODUCTION

In 2003 BaBar Collaboration discovered a positive-parity scalar charm strange meson $D_{s0}(2317)$ with a very narrow width [1], which was confirmed by CLEO Collaboration [2] and BELLE Collaboration [3] later. This observed state has attracted much attention because its measured mass and width do not match the predictions from potential-based quark models [4]. To resolve the difficulties, many theoretical models have appeared in the literature. Various theoretical models, based on the $c\bar{s}$ quark structure, are suggested to explain the low mass and the narrow width for the $D_{s0}(2317)$ meson [5-10]. QCD sum rule analysis in [11, 12] supports the $c\bar{s}$ postulation of nature $D_{s0}(2317)$. Apart from the quark-antiquark interpretation, this state has been interpreted as a DK molecule [13], a $D_s\pi$ molecule [14], a $c\bar{s}q\bar{q}$ four-quark state [15], and a mixing of the conventional state and the four-quark state [16]. Also this state was investigated in the framework of chiral symmetry considerations [17].

In this work, we assume $D_{s0}(2317)$ meson as the $c\bar{s}$ state and study its parameters at finite temperature using QCD sum rules method [18]. The extending of this method to finite temperature has been made in the paper [19]. The thermal QCD sum rule method has been extensively used for studying thermal properties of both light and heavy mesons as a reliable and well-established method [20-24]. In the present work, we calculated the temperature behavior of mass and leptonic decay constant of $D_{s0}(2317)$ using Borel transform sum rules method.

THERMAL QCD SUM RULE FOR THE SCALAR CHARMS STRANGE MESON

The starting point for the sum rule analysis is the two-point thermal correlator

$$\Pi(q^2) = i \int d^4 x e^{i q \cdot x} \langle T[J(x)J^+(0)] \rangle,$$

(1)

where $J(x) = (m_c - m_s); \bar{s}(x)\bar{c}(x)$: is heavy-light quark current and has the quantum numbers of the $D_{s0}(2317)$ meson, $m_c$ and $m_s$ are charm and strange quark masses respectively. $s$ quark mass is not neglected throughout this work. First
let us calculate theoretical part of the correlator Eq. (1). Up to a subtraction polynomial, which depends on the large $q^2$ behavior, $\Pi(q^2)$ satisfies following dispersion relation [19, 21, 22]

$$\Pi(q^2) = \int ds \frac{\rho(s)}{s + Q^2} + \text{subtractions} , \quad (2)$$

where $\rho(q) = \frac{1}{\pi} \Im \Pi(q) \tanh \left( \frac{\beta q_0}{2} \right)$ is spectral density. After some calculations we find that perturbative part of the spectral density is given by

$$\rho_{\text{pert}}(q, T) = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\omega_1^2 - k^2 + \mathbf{q} \cdot \mathbf{k} - \omega_1 q_0 + m_e m_s}{\omega_1 \omega_2} \times \left[ (1-n_1-n_2)\delta(q_0 - \omega_1 - \omega_2) + (n_1-n_2)\delta(q_0 - \omega_1 + \omega_2) \right] \quad (3)$$

Here $\omega_1 = \sqrt{q^2 + m_e^2}$ and $\omega_2 = \sqrt{(k - q)^2 + m_s^2}$. Note that spectral density involves two pieces, one is called the annihilation term, $\rho_{a, \text{pert}}(s, T)$, which survives at $T = 0$. Other term is called scattering term, $\rho_{s, \text{pert}}(s, T)$, which vanishes at $T = 0$. As can be seen, delta function $\delta(q_0 - \omega_1 - \omega_2)$ in Eq. (3) gives the first branch cut, $q^2 \geq (m_e + m_s)^2$, which coincides with zero temperature cut and describes the standard threshold for particle decays. On the other hand, delta function $\delta(q_0 - \omega_1 + \omega_2)$ in Eq. (3) shows that an additional branch cut arises at finite temperature, $q^2 \leq (m_e - m_s)^2$, and this new branch cut corresponds to particle absorption from the medium. Therefore, delta functions $\delta(q_0 - \omega_1 - \omega_2)$ and $\delta(q_0 - \omega_1 + \omega_2)$ in Eq. (3) contribute in regions $(m_e + m_s)^2 + q^2 \leq q_0^2 \leq \infty$ and $0 \leq q_0^2 \leq q^2 + (m_e - m_s)^2$ respectively. Taking into account these contributions the annihilation and scattering parts of spectral function in the case $q = 0$ can be written as

$$\rho_{a, \text{pert}}(s, T) = \rho_0(s) \left[ 1 - n \left( \frac{\sqrt{s}}{2} \left( 1 + \frac{m_e^2 - m_s^2}{s} \right) \right) - n \left( \frac{\sqrt{s}}{2} \left( 1 - \frac{m_e^2 - m_s^2}{s} \right) \right) \right], \quad (4)$$

$$\rho_{s, \text{pert}}(s, T) = \rho_0(s) \left[ n \left( \frac{\sqrt{s}}{2} \left( 1 + \frac{m_e^2 - m_s^2}{s} \right) \right) - n \left( -\frac{\sqrt{s}}{2} \left( 1 - \frac{m_e^2 - m_s^2}{s} \right) \right) \right]. \quad (5)$$

Here $\rho_0(s)$ is the correlation function in the lowest order of perturbation theory at zero temperature and given by

$$\rho_0(s) = \frac{3(m_e - m_s)^2}{8\pi^2 s} q^2(s) v^3(s), \quad (6)$$

where $q(s) = s - (m_e - m_s)^2$ and $v(s) = (1 - 4m_e m_s/q(s))^{1/2}$.

The contribution of perturbative two-loop order $\alpha_s$ corrections to the spectral density in perturbation theory at zero temperature can be written as [26]:

307
\[ \rho_1(s) = \frac{4\alpha_s}{3\pi} \rho_0(s) f(x), \quad (7) \]

where \( x = m_c^2/s \), \( \alpha_s = \alpha_s(m_c^2) \) and

\[ f(x) = \frac{9}{4} + 2Li_2(x) + \ln x \ln(1-x) - \frac{3}{2} \ln(1/x-1) - \ln(1-x) + x \ln(1/x-1) - \frac{x}{1-x} \ln x. \quad (8) \]

Here \( Li_2(x) = -\int_0^x \frac{\ln(1-t)}{t} dt \) is the dilogarithm function. Note that in \( \alpha_s \) corrections terms the strange quark mass is set zero, though in numerical analysis, the mass of the strange quark is taken account. The subtraction terms in Eq. (2) are removed by using Borel transformation, therefore we will omit these terms. The non-perturbative contributions at zero temperature to the correlator has following form

\[ \Pi_{np}(Q^2) = m_c \lambda \langle 0|\bar{s}s|0 \rangle \left[ 1 - \frac{1}{2} e(3-\lambda) - \lambda e^2 (1-\lambda) + \frac{1}{2} \alpha_s^3 \left( 1 + \frac{1}{2} \ln(\frac{\alpha_s}{\pi}) \right) \right] \]

\[ + \frac{\langle 0|\bar{s}s|0 \rangle}{12\pi} \left[ 1 - 3e \left( 1 - \frac{8}{3} \lambda + 2\lambda^2 - 2\lambda (1-\lambda) \ln(\frac{\alpha_s}{\pi}) \right) \right], \quad (9) \]

\[ + \frac{M_0^2}{2m_c} \langle 0|\bar{s}s|0 \rangle \lambda^2 (1-\lambda)(1-e(2-\lambda)) - \frac{8}{27} \frac{\pi\lambda}{m_c^2} \lambda^2 \left( 2 - \lambda - \lambda^2 \right) \]

which arise in the framework of the operator product expansion and parameterized by vacuum expectation values of quark and gluon fields in the QCD Lagrangian. In Eq. (9) \( \lambda = m_c^2 / (Q^2 + m_c^2) \), \( \epsilon = m_c / m_c \) and terms are organized according to their dimension. It is assumed that the expansion (9) also remains valid at finite temperature, but the vacuum condensates must be replaced by their thermal expectation values \[19\].

Our next task is the calculation of the physical part of the correlator (1). According to the basic idea of quark-hadron duality assumption, the right-hand side of Eq. (1) can be evaluated in a hadron-based picture. Equating operator product expansion and hadron representations of correlation function and using quark-hadron duality the central equation of our sum-rule analysis takes the form:

\[ \frac{f^2(T) m^4(T)}{Q^2 + m^2(T)} = \int_{(m_c, m_s)}^{s_0(T)} ds_0 \frac{P_{a,\text{per}}(s, T) + \rho_1(s)}{s + Q^2} + \int_0^{(m_c, m_s)} ds \frac{P_{a,\text{per}}(s, T)}{s + Q^2} + \Pi_{np}(Q^2, T), \quad (10) \]

where \( f \) and \( m \) are the leptonic decay constant and mass of \( D_{s0}(2317) \) meson respectively. Note that in Eq. (10) the mass and leptonic decay constant were replaced by their temperature dependent values. The continuum threshold also depends on temperature; to a very good approximation its scales universally as the quark condensate \[23\]

\[ s_0(T) = s_0 \frac{\langle q\bar{q} \rangle}{\langle 0|\bar{q}q|0 \rangle} \left[ 1 - \frac{(m_c + m_s)^2}{s_0} \right] + (m_c + m_s)^2. \quad (11) \]

where in the right hand side \( s_0 \) is hadronic threshold at zero temperature: \( s_0 = s(T = 0) \).
NUMERICAL ANALYSIS OF MASS AND LEPTONIC DECAY CONSTANT

In this section we present our results for the temperature dependence of $D_{s0}(2317)$ meson mass and leptonic decay constant. Performing Borel transformation with respect to $Q_0^2$ from both sides of equation (10) and taking the derivative with respect to $1/M^2$ from both sides of obtained expression, and making some transformations we have

$$m^2(T) = B(T)/A(T),$$  \hspace{1cm} (12)

$$f^2(T) = \frac{A(T)}{m^2(T)} \exp\left(\frac{m^2(T)}{M^2}\right),$$  \hspace{1cm} (13)

where

$$A(T) = \int_{(m_s - m_c)^2}^{s_f} ds \left[\rho_{u,\text{pert}}(s) + \rho_{c}(s)\right] \exp\left(-\frac{s}{M^2}\right) + \int_0^{(m_s - m_c)^2} ds \rho_{s,\text{pert}}(s) \exp\left(-\frac{s}{M^2}\right) + \Pi_{\text{np}}(M^2, T),$$  \hspace{1cm} (14)

$$\Pi_{\text{np}}(M^2, T) = m_c^3 \langle \bar{s}s \rangle e^{-\beta} \left[1 - \frac{3}{2} \frac{e}{\beta} + \frac{1}{2} \beta e - \beta e^2 \left(1 - \frac{1}{2} \beta\right) + \frac{1}{2} e^3 \left(1 + \beta - 2 \beta^2 + \frac{1}{3} \beta^3\right)\right]$$

$$+ \frac{1}{12} \frac{\alpha_s G_F^2}{\pi} m_c^2 e^{-\beta} \left[1 - 3 e \left(1 - \frac{8}{3} \beta + \beta^2 - 2 \beta \ln(\beta e) + \gamma - 1\right) + \beta^2 \left(\ln(\beta e) + \gamma - \frac{3}{2}\right)\right],$$

$$+ \frac{1}{2} M_0^2 m_c \beta \langle \bar{s}s \rangle e^{-\beta} \left[1 - \frac{1}{2} \beta - 2 e \left(1 - \frac{3}{4} \beta \left(1 - \frac{1}{9} \beta\right)\right)\right] - \frac{4}{81} \pi \alpha_s \rho(\bar{s}s) \beta e^{-\beta} \left(12 - 3 \beta - \beta^2\right)$$

where $\beta = m_c^2 / M^2$ and $B(T) = -m_c^2 \frac{dA(T)}{d\beta}$.

FIGURE 1. Temperature dependences of meson mass and leptonic decay constant at $s_0 = 8 \text{ GeV}^2$. 

309
For the numerical evolution of the above sum rule, we use standard QCD input parameters [12, 18, 27]. The criterion we adopt here is to fix $s_0$ in such a way so as to reproduce the zero temperature values of meson mass and leptonic decay constant. $D_{e\bar{e}}(2317)$ meson mass and leptonic decay constant as a function of temperature are shown in Fig.1 at continuum threshold value $s_0 = 8.0 GeV^{-2}$. As seen, mass decreases with increasing temperature and mesons lose approximately 15% of their mass at $T = 150 MeV$ temperature. Also, leptonic decay constant $f$ decreases with increasing temperature and vanishes approximately at critical temperature. This situation may be interpreted as a signal for deconfinement and agrees with light and heavy mesons investigations [22-24]. Numerical analysis shows that the temperature dependence of $f$ is the same, when $M^2$ changes between 1.5$GeV^{-2}$ and 3$GeV^{-2}$ at fixed values of continuum threshold. Obtained results can be used for interpretation heavy ion collision experiments.

ACKNOWLEDGEMENTS

The authors have much pleasure to thank T.M. Aliev and A. Özpineci for useful discussions. This work is supported by the Scientific and Technological Research Council of Turkey (TUBITAK), research project No.105T131.

REFERENCES