Abstract

In the present work, we investigate properties of $B_c$ meson in the basis of thermal QCD sum rules. The annihilation and scattering parts of spectral density are calculated in the lowest order of perturbation theory. Taking into account the additional operators arising at finite temperature and perturbative two-loop order $\alpha_s$ correction to the spectral density, thermal QCD sum rule is obtained. In particular, the temperature dependence of the mass and leptonic decay constant of $B_c$ mesons are discussed. The obtained results show that at critical or deconfinement temperature, the decay constant decreases approximately 54%. The results at zero temperature are in a good consistency with the existing experimental values as well as predictions of the other nonperturbative approaches.

Keywords: Thermal QCD sum rules, $B_c$ meson, leptonic decay constant

1. Introduction

Because of its flavor charge, the $B_c$ meson, that is firstly explored by CDF collaboration [1] in the $B_c \rightarrow J/\Psi \ell \nu$ decay at $\sqrt{s} = 1.8$ TeV $pp$ collision, achieves significant breakthrough in the search for dynamics of the heavy quarks. Considering its many properties, the $B_c$ meson as the ground state of $b\bar{c}$ system, is among the charmonium and bottomonium systems. It has a reach spectroscopy considering the orbital and angular momentum excitations. Different from the $J/\Psi$ and $\Upsilon$ states, the $B_c$ meson decays via only weak interactions, consequently it has a measurable life time. Because of two heavy quarks, the $B_c$ meson has a more reach decay channels compared to the other $B$ mesons. This is the reason why this meson has been in focus of much attention both theoretically and experimentally. This meson provides reliable determination of the $V_{cb}$ as an element of the CKM matrix and understanding new physics beyond the standard model.

Investigation of hadronic properties requires some nonperturbative approaches. One of most attractive and applicable nonperturbative approaches is the QCD sum rule [2, 3]. This method has been extensively used as an efficient tool for investigation of hadron properties in vacuum [4, 5]. QCD sum rules method has been extended to the finite temperature in [6]. The thermal QCD sum rules has some new features [7]-[10]. One of the new aspects is interaction of the particles existing in the medium with the currents which requires modification of the hadronic spectral function. The other new aspect of is breakdown of the Lorentz invariance via the
choice of reference frame. Due to residual $O(3)$ symmetry, more operators with the same dimensions appear in the OPE comparing to the QCD sum rules in vacuum.

Our aim in this work is to investigate the temperature dependence of leptonic decay constant $f_{B_c}$ of the pseudoscalar $B_c$ meson, which is defined by vacuum to meson matrix element of the axial vector current as:

\[
0 \langle 0|\bar{b}g_\gamma\gamma_5c|B_c \rangle = i f_{B_c} q_\mu .
\]

Taking into account the thermal spectral density and additional operators arising at finite temperature, the thermal behavior of decay constant is discussed. Analysis of parameters of the heavy mesons with respect to temperature can give valuable information about the QCD vacuum and transition to the quark gluon plasma (QGP) phase [10]-[15].

2. Thermal QCD Sum Rules

We start considering the two-point thermal correlation function,

\[
\Pi(q, T) = i \int d^4 x e^{i q x} \langle 0| J(x) J(0) |0 \rangle,
\]

where $J(x) = \bar{b}(x)\gamma_5c(x)$ is interpolating current for $B_c$ meson. To obtain sum rules for physical observables, we need to calculate the correlation function in two different ways. In QCD or theoretical side, the correlation function is calculated in deep Euclidean region, $q^2 \ll -M_{B_c}^2$ via OPE where the short and long distance contributions are separated,

\[
\Pi^{QCD}(q, T) = \int \frac{d^4 p}{(2\pi)^4} \frac{\rho_{pert}(s, T)}{s - q^2} + \Pi^{nonpert}(q, T),
\]

where $\rho(s, T)$ is called the thermal spectral density and at fixed $|q|$ it is written as:

\[
\rho(q, T) = \frac{1}{\pi} Im \Pi^{pert}(q, T) \tanh \left( \frac{B_\rho}{2} \right).
\]

Using the quark propagator at finite temperature, the imaginary part of the perturbative correlation function is obtained as:

\[
\Pi^{pert}(q, T) = 4iN_c \int \frac{d^4 k}{(2\pi)^4} (k^2 - k \cdot q - m_b m_c) \times D(k, m_b)D(k - q, m_c),
\]

where $D(k) = 1/(k^2 - m^2 + i\epsilon) + 2\pi i n(|k_0|) \delta(k^2 - m^2)$ and $n(x) = \exp(x) + 1^{-1}$ is the Fermi distribution function. The $\rho(q, T)$ compose of two parts, namely annihilation and scattering parts of spectral density satisfying the $q^2 \geq (m_b + m_c)^2$ and $q^2 \leq (m_b - m_c)^2$ conditions, respectively. The annihilation part which coincides with zero temperature cut, describes the standard threshold for particle decays. On the other hand, the scattering part arising only at finite temperature, corresponds to particle absorption from the medium. After some simplifications, these parts of spectral density in the case $q = 0$ can be written as:

\[
\rho^{pert}(s, T) = \rho_{0}(s) \left[ 1 - f(s, m_b, m_c) \right],
\]

for $(m_b + m_c)^2 \leq s \leq \infty$, and

\[
\rho^{nonpert}(s, T) = \rho_{s}(s) \left[ f(s, m_b, m_c) - f(-s, m_b, m_c) \right],
\]

for $0 \leq s \leq (m_b - m_c)^2$. Here $f(s, m_b, m_c) = n(\sqrt{s + m_b^2 - m_c^2}/(2\pi))$ and $\rho_{s}(s)$ is the spectral density in the lowest order of perturbation theory at zero temperature and is given by:

\[
\rho_{s}(s) = \frac{3}{8\pi^2}q^2(s)v(s),
\]

where $q(s) = s - (m_b - m_c)^2$ and $v(s) = \sqrt{1 - 4m_b m_c}/q(s)$. When doing the numerical calculations at $T = 0$, the contribution coming from two-loop diagrams $\rho_{\alpha}$ [4, 16] are also taken into account, but since its expression is very lengthy, we do not present its explicit expression here.

To calculate the nonperturbative part in QCD side, we use the nonperturbative part of the quark propagator in an external gluon field, $A^{\mu}_{0}(s)$ in the Fock-Schwinger gauge, $\partial^{\mu}A^{\mu}_{0}(x) = 0$. Taking into account one and two gluon lines attached to the quark line, the massive quark propagator up to terms necessary for our calculations can be written as:

\[
S^{\mu\nu}_{\text{nonpert}}(k) =
\]

\[
- \left( \frac{i}{8} \delta(\gamma^5) - \frac{G_{\epsilon\lambda}}{(k^2 - m^2)^2} \right) \left[ \sigma_{\epsilon\lambda}(k + m) + (k + m)\sigma_{\epsilon\lambda} \right] + \frac{G_{\epsilon\lambda} G^{\epsilon\lambda}}{4(k^2 - m^2)^2} \left( 3m(k^2 + m k) \right) + \frac{G_{\epsilon\lambda} G^{\epsilon\lambda}}{4} \left( m^2 - 4(k \cdot u)^2 \right) \frac{k}{k} + 4(k \cdot u)(k^2 - m^2) \left( u^\mu \Theta_{\alpha\beta}^\mu \phi \right),
\]
where, $u^a$ is the four-velocity of the heat bath and $\Theta^{a\beta}_{\text{eff}}$ is the traceless, gluonic part of the energy-momentum tensor of the QCD.

Now, we turn our attention to calculate the physical or phenomenological side of the correlation function. The hadronic spectral density is expressed by the ground state pseudoscalar meson pole plus the contribution of the higher states and continuum:

$$
\rho{s}^{\text{had}}(s) = \frac{f_R^2(T) m_B^4(T)}{(m_B + m_c)^2} s(s - m_B^2) + \Theta(s - s_0) \rho^{\text{pert}}(s) \tag{9}
$$

Matching the phenomenological and QCD sides of the correlation function, sum rules for the mass and decay constant of pseudoscalar meson are obtained. Performing Borel transformation over the $Q^2_0 = -q^2$ after lengthy calculations, we obtain the following sum rule for $B_c$ mesons:

$$
f_R^2(T) m_B^4(T) \exp\left(\frac{m_B^2}{s}\right) = (m_b + m_c)^2 \times \left\{ \int_{m_B + m_c}^{\text{had}} (\rho^{\text{pert}}(s) + \rho_{\text{eff}}(s)) \exp\left(-\frac{s}{M^2}\right) \right\}
$$

where $M^2$ is the Borel mass parameter and $\tilde{\Pi}^{\text{nonpert}}$ shows the nonperturbative part of QCD side in Borel transformed scheme:

$$
\tilde{\Pi}^{\text{nonpert}} = \int_0^1 \frac{dx}{1} \frac{1}{96 \pi M^6} x^4 (-1 + x)^3 \exp\left[\frac{m_c^2 x - m_c^2 (1 + x)}{M^2 x (1 + x)}\right] \mathbb{E}\left\{\alpha_s G^2\right\} - m_B^2 (-1 + x)^6 + m_b m_c x (-1 + x)^2 (1 - 2 x) + m_b^2 x (-1 + x)^2 (1 - 2 x) + 4 M^4 (-1 + x)^2 \times (2 - x + x^2 + M^2 (-4 + 3 x + 5 x^2 - 4 x^3)) - m_B^2 x (-1 + x)^2 (1 - 3 x + x^2) + 2 M^2 (1 - 2 x + x^2) - m_B^2 x (-1 + x)^2 (1 - 3 x + x^2) + 2 M^2 (1 - 2 x + x^2) \times (5 - 17 x + 24 x^2 - 12 x^3) + M^4 (-1 + x)^2 (-1 + 15 x - 7 x^2 + 2 x^3) + x^3 (m_c^3 x + M^2 (-1 + x)^3 (9 - 11 x + 11 x^2)) + 2 m_b^2 M^6 x (-1 + 4 x - 4 x^2 + x^3) - m_B^2 M^4 \times (-1 + x)^2 (-9 + 7 x + x^2 + 2 x^3) + \int_0^1 dx \rho^{\text{pert}}(s) \exp\left(-\frac{s}{M^2}\right)
$$

In the rest frame of the heat bath, the results of some observables calculated using lattice QCD in [17]-[19] are fitted well by the following parametrization for the thermal average of total energy density, $\langle \Theta \rangle$:

$$
\langle \Theta \rangle = 2 \langle \Theta^0 \rangle = 6 \times 10^{-6} \exp\left[80 (T - 0.1)\right] \times \langle GeV^4\rangle, \tag{12}
$$

where temperature $T$ is measured in units of GeV and this parametrization is valid only in the interval $0.1 \text{GeV} \leq T \leq 0.17 \text{GeV}$. Here, we would like to stress that the total energy density has been calculated for $T \geq 0$ in chiral perturbation theory [20], while this quantity has only been obtained for $T > 100 \text{MeV}$ in lattice QCD.

In our calculation we use the temperature dependent continuum threshold, $s_0(T)$, gluon condensate, $(\langle G^2 \rangle)$, and strong coupling constant, as presented in [14]. The input parameters are taken to be the standard values $m_c = (1.3 \pm 0.5) \text{GeV}$, $m_b = (4.7 \pm 0.1) \text{GeV}$ and $(0 | \frac{1}{2} \alpha_s G^3 | 0) = (0.012 \pm 0.004) \text{GeV}^4$ for quark masses and gluon condensate at zero temperature. We choose the value $44 \text{GeV}^2 \leq s_0 \leq 46 \text{GeV}^2$ for the continuum threshold and work at region for the Borel mass parameter, $M^2 = 10 \text{GeV}^2 \leq M^2 \leq 25 \text{GeV}^2$.

We present the dependence of mass and decay constant on the temperature, $T$ in Fig.1. The figure depict that both parametrization of lattice QCD and chiral perturbation theory predict the same result in validation limit of lattice QCD fit parametrization, i.e., $0.10 \text{GeV} \leq T \leq 0.17 \text{GeV}$. These results show that the masses and decay constants remain unchanged approximately up to $T \geq 100 \text{MeV}$, but after this point, they start to diminish with increasing the temperature.
At \( T = 0 \), the mass and leptonic decay constant of \( B_c \) meson are obtained as \( m_{B_c} = (6.367 \pm 0.038) \text{ GeV} \) and \( f_{B_c} = (0.476 \pm 0.0265) \text{ GeV} \). These results are in good consistency with the existing experimental data and predictions of other nonperturbative models [21]-[23]. Our results for the leptonic decay constant of \( B_c \) at zero temperature as well as the behavior of the mass and decay constant with respect to the temperature can be checked in the future experiments. The obtained behavior of the observables in terms of temperature can be used in analysis of the results of the heavy ion collision experiments.

Figure 1: **Up Panel:** The dependence of the mass of \( B_c \) meson in GeV on temperature at \( M^2 = 20 \text{ GeV}^2 \). **Down Panel:** The dependence of the leptonic decay constant of \( B_c \) meson in GeV on temperature at \( M^2 = 20 \text{ GeV}^2 \).

References