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Intuitionistic fuzzy soft modules

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ABSTRACT

Molodtsov (1999) initiated the concept of soft sets in [1]. Maji et al. (2003) defined some operations on soft sets in [22]. Aktaş and Çağman (2007) generalized soft sets by defining the concept of soft groups in [16]. After them, Sun et al. (2008) gave soft modules in [21]. In this paper, the concept of an intuitionistic fuzzy soft module is introduced and some operations on intuitionistic fuzzy soft sets are given. Finally, some of its basic properties are studied.

1. Introduction

Many practical problems in economics, engineering, environment, social science, medical science etc. cannot be dealt with by classical methods, because classical methods have inherent difficulties. The reason for these difficulties may be due to the inadequacy of the theories of parameterization tools. Molodtsov [1] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. After Molodtsov's work, some different applications of soft sets were studied in [2]. Maji et al. [3] presented the concept of fuzzy soft sets. The theory of fuzzy sets, first developed by Zadeh in [4], is perhaps the most appropriate framework for dealing with uncertainties. Rosenfeld [5] proposed the concept of fuzzy groups in order to establish the algebraic structures of fuzzy sets. Several researchers have studied fuzzy modules, for example, [6–12]. The concept of intuitionistic fuzzy sets which is a generalization of fuzzy sets was introduced by Atanassov in [13]. Davvaz [14] defined the concept of intuitionistic fuzzy module by using intuitionistic fuzzy sets. Gunduz (Aras) and Davvaz in [15] did some investigations on intuitionistic fuzzy modules. Aktaş and Çağman [16] defined soft groups and compared soft sets with fuzzy sets and rough sets. Feng et al. [17] gave soft semirings and Acar et al. [18] introduced initial concepts of soft rings. After that the definition of fuzzy soft groups was given by some authors [19,20]. Qiu-Mei Sun et al. [21] defined soft modules and investigated their basic properties.

The main purpose of this paper is to introduce a basic version of intuitionistic fuzzy soft module theory, which extends the notion of modules by including some algebraic structures in soft sets. Finally, we investigate some basic properties of intuitionistic fuzzy soft modules.

2. Preliminaries

In this section, we recall some basic concepts of fuzzy soft set theory. Let E be a convenient parameter set for the universe X.

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Definition 2.1 ([22]). Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if and only if F is a mapping from E into the set of all subsets of the set X, i.e., $F : E \to P(X)$, where $P(X)$ is the power set of X.

In other words, the soft set is a parameterized family of subsets of the set X. Every set $F(e)$, for every $e \in E$, may be considered as the set of e-elements of the soft set (F, E), or as the set of e-approximate elements of the soft set.

In this manner, a soft set (F, E) is given as consisting of a collection of approximations:

$$(F, E) = \{F(e) : e \in E\}.$$

Definition 2.2 ([3]). Let I^X denote the set of all fuzzy sets on X and $A \subset E$. A pair (f, A) is called a fuzzy soft set over X, where f is a mapping from A into I^X. That is, for each $a \in A$, $f(a) = f_a : X \to I$, is a fuzzy set on X.

Definition 2.3 ([3]). For two fuzzy soft sets (f, A) and (g, B) over a common universe X, we say that (f, A) is a fuzzy soft subset of (g, B) and write $(f, A) \subseteq (g, B)$ if

(i) $A \subset B$, and
(ii) For each $a \in A$, $f_a \leq g_a$, that is, f_a is fuzzy subset of g_a.

Definition 2.4 ([3]). Two fuzzy soft sets (f, A) and (g, B) over a common universe X are said to be equal if $(f, A) \subseteq (g, B)$ and $(g, B) \subseteq (f, A)$.

Definition 2.5 ([3]). Union of two fuzzy soft sets (f, A) and (g, B) over a common universe X is the fuzzy soft set (h, C), where $C = A \cup B$ and

$$h(c) = \begin{cases} f_c, & \text{if } c \in A - B \\ g_c, & \text{if } c \in B - A \\ f_c \vee g_c, & \text{if } c \in A \cap B, \end{cases} \qquad \forall c \in C.$$

It is denoted as $(f, A) \cup (g, B) = (h, C)$.

Definition 2.6 ([3]). Intersection of two fuzzy soft sets (f, A) and (g, B) over a common universe X is the fuzzy soft set (h, C), where $C = A \cap B$ and $h_c = f_c \wedge g_c$, $\forall c \in C$.

It is written as $(f, A) \cap (g, B) = (h, C)$.

Definition 2.7 ([3]). If (f, A) and (g, B) are two soft sets, then (f, A) **AND** (g, B) is denoted as $(f, A) \wedge (g, B)$. $(f, A) \wedge (g, B)$ is defined as $(h, A \times B)$ where $h(a, b) = h_{a,b} = f_a \wedge g_b$, $\forall (a, b) \in A \times B$.

Now, let M be a left R-module, A be any nonempty set. $F : A \to P(M)$ refers to a set-valued function and the pair (F, A) is a soft set over M.

Definition 2.8 ([21]). Let (F, A) be a soft set over M. (F, A) is said to be a soft module over M if and only if $F(x) < M$ for all $x \in A$.

Definition 2.9 ([21]). Let (F, A) and (G, B) be two soft modules over M and N respectively. Then $(F, A) \times (G, B) = (H, A \times B)$ is defined as $H(x, y) = F(x) \times G(y)$ for all $(x, y) \in A \times B$.

Proposition 2.10 ([21]). *Let (F, A) and (G, B) be two soft modules over M and N respectively. Then $(F, A) \times (G, B)$ is soft module over $M \times N$.*

Definition 2.11 ([21]). Let (F, A) and (G, B) be two soft modules over M and N respectively, $f : M \to N$, $g : A \to B$ be two functions. Then we say that (f, g) is a soft homomorphism if the following conditions are satisfied:

(1) f is a homomorphism from M onto N,
(2) g is a mapping from A onto B, and
(3) $f(F(x)) = G(g(x))$ for all $x \in A$.

Definition 2.12 ([19]). Let (F, A) be a fuzzy soft set over G. Then (F, A) is said to be a fuzzy soft group over G if and only if $F(x)$ is a fuzzy subgroup of G, for all $x \in A$.

Theorem 2.13 ([19]). *Let (F, A) and (H, A) be two fuzzy soft groups over G. Then their intersection $(F, A) \tilde{\cap} (H, A)$ is a fuzzy soft group over G.*

Theorem 2.14 ([19]). *Let (F, A) and (H, B) be two fuzzy soft groups over G. If $A \cap B = \emptyset$, then $(F, A) \tilde{\cup} (H, B)$ is a fuzzy soft group over G.*

Theorem 2.15 (*[19]*). *Let* (F, A) *and* (H, B) *be two fuzzy soft groups over G. Then* $(F, A) \wedge (H, B)$ *is a fuzzy soft group over G.*

Definition 2.16 (*[13]*). An intuitionistic fuzzy set A in a non-empty set X is an object having the form

$$A = \{(x, \mu_A(x), \lambda_A(x)) \, | x \in X\},$$

where the functions $\mu_A : X \to [0, 1]$ and $\lambda_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \lambda_A)$ for the intuitionistic fuzzy set $\{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$.

Definition 2.17 (*[14]*). Let M be a module over a ring R. An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ in M is called an intuitionistic fuzzy submodule of M if

(1) $\mu_A(0) = 1$,
(2) $\min\{\mu_A(x), \mu_A(y)\} \leq \mu_A(x - y)$ for all $x, y \in M$,
(3) $\mu_A(x) \leq \mu_A(r.x)$ for all $x \in M$ and $r \in R$,
(4) $\lambda_A(0) = 0$,
(5) $\lambda_A(x - y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$ for all $x, y \in M$,
(6) $\lambda_A(r.x) \leq \lambda_A(x)$ for all $x \in M$ and $r \in R$.

Definition 2.18 (*[15]*). The map $f : (M, \mu, \lambda) \to (M', \mu', \lambda')$ is a homomorphism of intuitionistic fuzzy modules if and only if the conditions $\mu'(f(x)) \geq \mu(x)$ and $\lambda'(f(x)) \leq \lambda(x)$ are satisfied.

3. Intuitionistic fuzzy soft modules

In this section, we firstly define an intuitionistic fuzzy soft set and give some operations on this set. Let X be an initial universe set and IFS(X) denote the family of intuitionistic fuzzy sets on X.

Definition 3.1. Let IFS(X) denote the set of all intuitionistic fuzzy sets on X and $A \subset E$. A pair (F, A) is called an intuitionistic fuzzy soft set over X, where F is a mapping from A into IFS(X). That is, for each $a \in A$, $F(a) = (F_a, F^a) : X \to I$ is an intuitionistic fuzzy set on X, where $F_a, F^a : X \to I$ are fuzzy sets.

Definition 3.2. For two intuitionistic fuzzy soft sets (F, A) and (G, B) over a common universe X, we say that (F, A) is an intuitionistic fuzzy soft subset of (G, B) and write $(F, A) \subseteq (G, B)$ if

(i) $A \subset B$, and
(ii) For each $a \in A$, $F_a \leq G_a$, $F^a \geq G^a$.

Definition 3.3. Two intuitionistic fuzzy soft sets (F, A) and (G, B) over a common universe X are said to be equal if $(F, A) \subseteq (G, B)$ and $(G, B) \subseteq (F, A)$.

Definition 3.4. Union of two intuitionistic fuzzy soft sets (F, A) and (g, B) over a common universe X is the intuitionistic fuzzy soft set (H, C), where $C = A \cup B$ and

$$H(c) = \begin{cases} (F_c, F^c), & \text{if } c \in A - B \\ (G_c, G^c), & \text{if } c \in B - A \\ (F_c \vee G_c, F^c \wedge G^c) & \text{if } c \in B \cap A, \end{cases} \qquad \forall c \in C.$$

It is denoted as $(F, A) \cup (G, B) = (H, C)$.

Definition 3.5. The intersection of two intuitionistic fuzzy soft sets (F, A) and (G, B) over a common universe X is the intuitionistic fuzzy soft set (H, C), where $C = A \cap B$ and $H(c) = (F_c \wedge G_c, F^c \vee G^c)$, $\forall c \in C$.
It is written as $(F, A) \cap (G, B) = (H, C)$.

Definition 3.6. If (F, A) and (G, B) are intuitionistic two soft sets, then (F, A) **AND** (G, B) is denoted as $(F, A) \wedge (G, B)$. $(F, A) \wedge (G, B)$ is defined as $(H, A \times B)$ where $H(a, b) = (F_a \wedge G_b, F^a \vee G^b)$, $\forall (a, b) \in A \times B$.

In this paper R is an ordinary ring. Let M be a left (or right) R- module, and let $A \neq \emptyset$ be a set. IFS(M) denotes the family of intuitionistic fuzzy sets over M.

Definition 3.7. Let (F, A) be an intuitionistic fuzzy soft set over M. Then (F, A) is said to be an intuitionistic fuzzy soft module over M iff $\forall a \in A$, $F(a) = (F_a, F^a)$ is an intuitionistic fuzzy submodule of M.

Definition 3.8. Let (F, A) and (H, B) be two intuitionistic fuzzy soft modules over M and N respectively, and let $f : M \to N$ be a homomorphism of modules, and let $g : A \to B$ be a mapping of sets. Then we say that $(f, g) : (F, A) \to (H, B)$ is an intuitionistic fuzzy soft homomorphism of intuitionistic fuzzy soft modules, if the following conditions are satisfied:

$$f(F_a) = H_{g(a)}, \qquad f(F^a) = H^{g(a)}.$$

We say that (F, A) is an intuitionistic fuzzy soft homomorphic to (H, B).

Note that for $\forall a \in A, f : (M, F_a, F^a) \to (N, H_{g(a)}, H^{g(a)})$ is an intuitionistic fuzzy homomorphism of intuitionistic fuzzy modules [15].

Intuitionistic fuzzy soft modules and their morphisms consist of a category. This category is denoted IFSM.

Theorem 3.1. *Let (F, A) and (H, B) be two intuitionistic fuzzy soft modules over M. Then their intersection $(F, A) \cap (H, B)$ is an intuitionistic fuzzy soft module over M.*

Proof. Let $(F, A) \cap (H, B) = (G, C)$, where $C = A \cap B$. Since the intuitionistic fuzzy soft set $(G_c, G^c) = (F_c \wedge H_c, F^c \vee H^c)$ is an intuitionistic fuzzy submodule, for $\forall c \in C$, (G, C) is an intuitionistic fuzzy soft module over M. □

Theorem 3.2. *Let (F, A) and (H, B) be two intuitionistic fuzzy soft modules over M. Then $(F, A) \wedge (H, B)$ is an intuitionistic fuzzy soft module over M.*

Proof. By Definition 2.7, we can write $(F, A) \wedge (H, B) = (G, A \times B)$. Since (F_a, F^a) and (H_b, H^b) are intuitionistic fuzzy submodules of M, $(F_a \wedge H_b, F^a \vee H^b)$ is an intuitionistic fuzzy submodule of M. Thus, $G(a, b) = \left(F_a \wedge H_b, F^a \vee H^b\right)$ is an intuitionistic fuzzy submodule of M, for all $(a, b) \in A \times B$. Hence, we find that $(F, A) \wedge (H, B)$ is an intuitionistic fuzzy soft module over M. □

Theorem 3.3. *Let (F, A) and (H, B) be two intuitionistic fuzzy soft modules over M. If $A \cap B = \emptyset$, then $(F, A) \cup (H, B)$ is a intuitionistic fuzzy soft module over M.*

Proof. By Definition 2.5, we can write $(F, A) \cup (H, B) = (G, C)$. Since $A \cap B = \emptyset$, it follows that either $c \in A - B$ or $c \in B - A$ for all $c \in C$. If $c \in A - B$, then $G(b) = (F_b, F^b)$ is an intuitionistic fuzzy submodule of M, and if $c \in B - A$, then $G(b) = (H_b, H^b)$ is an intuitionistic fuzzy submodule of M. Hence, $(F, A) \cup (H, B)$ is an intuitionistic fuzzy soft module over M. □

Definition 3.9. Let (F, A) and (H, B) be two intuitionistic fuzzy soft modules over M. Then (F, A) is called an intuitionistic fuzzy soft submodule of (H, B) if

(1) $A \subset B$.
(2) For all $a \in A$, (F_a, F^a) is an intuitionistic fuzzy submodule of (H_a, H^a).

Theorem 3.4. *Let (F, A) and (H, A) be two intuitionistic fuzzy soft modules over M. If $F(a) \leq H(a)$ for all $a \in A$, then (F, A) is an intuitionistic fuzzy soft submodule of (H, A).*

Proof. The proof of the theorem is straightforward. □

The following theorem is a generalized form of Theorems 3.1–3.3.

Theorem 3.5. *Let (F, A) be an intuitionistic fuzzy soft module over M, and let $\{(F_i, A_i)\}_{i \in I}$ be nonempty family of intuitionistic fuzzy soft submodules of (F, A). Then*

(1) $\prod_{i \in I}(F_i, A_i)$ *is an intuitionistic fuzzy soft submodule of (F, A),*
(2) $\wedge_{i \in I}(F_i, A_i)$ *is an intuitionistic fuzzy soft submodule of (F, A),*
(3) *If $A_i \cap A_j = \emptyset$, for all $i, j \in I$, then $\vee_{i \in I}(F_i, A_i)$ is an intuitionistic fuzzy soft submodule of (F, A).*

Let (F, A) and (H, B) be two intuitionistic fuzzy soft modules over M and N respectively, and $(f, g) : (F, A) \to (H, B)$ be an intuitionistic fuzzy soft homomorphism of these modules.

Now in this section we introduce the kernel and image of fuzzy soft homomorphism of intuitionistic fuzzy soft modules. Let $M' = \ker f$. Define $F' : A \to \mathrm{IFS}(M')$ by $F'_a = F_a \mid'_M, F'^a = F^a \mid'_M$. Then (F', A) is an intuitionistic fuzzy soft module over M'. It is clear that this module is an intuitionistic fuzzy soft submodule of (F, A).

Definition 3.10. (F', A) is said to be kernel of (f, g) and denoted by $\ker(f, g)$.

Now, let $B' = g(A)$. Then for all $b \in B'$, there exists $a \in A$ such that $g(a) = b$. Let $N' = \mathrm{Im} f < N$. We define the mapping $H' : B' \to \mathrm{IFS}(N')$ as $H'(b') = (H_{g(a)} \mid'_N, H^{g(a)} \mid'_N)$. Since (f, g) is an intuitionistic fuzzy soft homomorphism, $f(F_a) = H_{g(a)}, f(F^a) = H^{g(a)}$ is satisfied for all $a \in A$. Then the pair (H', B') is an intuitionistic fuzzy soft module over N' and (H', B') is an intuitionistic fuzzy soft submodule of (H, B).

Definition 3.11. (H', B') is said to be the image of (f, g) and denoted by $\mathrm{Im}(f, g)$.

Proposition 3.6. *Let (F, A) be an intuitionistic fuzzy soft module over M and N be an R-module and $f : M \to N$ be a homomorphism of R-modules. Then $(f(F), A)$ is an intuiitionistic fuzzy soft module over N.*

Proof. If the mapping $f(F) : A \to \mathrm{IFS}(N)$ is defined by

$$(f(F))_a(y) = \sup\{F_a(x) : f(x) = y\}, \qquad (f(F))^a(y) = \inf\left\{F^a(x) : f(x) = y\right\},$$

the proof is complete. □

Note that $(f, 1_A) : (F, A) \to (f(F), A)$ is an intuitionistic fuzzy soft homomorphism of intuitionistic fuzzy soft modules.

Proposition 3.7. *If M is an R-module, (H, A) is an intuitionistic fuzzy soft module over N and $f : M \to N$ is a homomorphism of R-modules, then $(f^{-1}(H), A)$ is an intuitionistic fuzzy soft module over M.*

Proof. If the mapping $f^{-1}(H) : A \to \mathrm{IFS}(M)$ is defined by

$$(f^{-1}(H))_a(x) = H_a(f(x)), \qquad (f^{-1}(H))^a(x) = H^a(f(x)),$$

the proof is complete. □

It is clear that $(f, 1_A) : (f^{-1}(H), A) \to (H, A)$ is an intuitionistic fuzzy soft homomorphism of intuitionistic fuzzy soft modules.

Lemma 3.8. *Let M and N be an R-modules and $f : M \to N$ be an R-homomorphism and (F, A) and (H, A) are two intuitionistic fuzzy soft modules over M and N respectively.*

(i) *$(f, 1_A) : (F, A) \to (H, A)$ is a intuitionistic fuzzy soft homomorphism if and only if for all $a \in A$, $H_a \geq f(F_a)$, $H^a \leq f(F^a)$ are satisfied.*
(ii) *$(f, 1_A) : (F, A) \to (H, A)$ is an intuitionistic fuzzy soft homomorphism if and only if for all $a \in A$, $F_a \leq f^{-1}(H_a)$, $F^a \geq f^{-1}(H^a)$ are satisfied.*

Now we define other algebraic operations over intuitionistic fuzzy soft modules. For this, we firstly give these algebraic operations over soft modules.

Let $\{(F_i, A_i)\}_{i\in I}$ be a family of soft modules over $\{M_i\}_{i\in I}$. Define

$$F : \prod_{i\in I} A_i \to \prod_{i\in I} M_i$$

by $F(\{a_i\}) = \prod_{i\in I} F(a_i)$. Since $\prod_{i\in I} F(a_i)$ is a submodule of $\prod_{i\in I} M_i$, $\left(F, \prod_{i\in I} A_i\right)$ is a soft module over $\prod_{i\in I} M_i$. This soft module is denoted as $\prod_{i\in I}(F_i, A_i)$.

Definition 3.12. $\prod_{i\in I}(F_i, A_i)$ is said to be direct product of soft modules.

It is clear that if $p_i : \prod_{i\in I} M_i \to M_i$ and $q_i : \prod_{i\in I} A_i \to A_i$ are projection mappings, then $(p_i, q_i) : \prod_{i\in I}(F_i, A_i) \to (F_i, A_i)$ is a soft homomorphism of soft modules.

Proposition 3.9. *Let $\{(F_i, A_i)\}_{i\in I}$ be a family of soft modules over $\{M_i\}_{i\in I}$ and $\{(H_i, B_i)\}_{i\in I}$ be a family of soft modules over $\{N_i\}_{i\in I}$ and $(f_i, g_i) : (F_i, A_i) \to (H_i, B_i)$ be a soft homomorphism of soft modules for each $i \in I$. Then $\left(\prod_{i\in I} f_i, \prod_{i\in I} g_i\right) : \prod_{i\in I}(F_i, A_i) \to \prod_{i\in I}(H_i, B_i)$ is a soft homomorphism of soft modules.*

Proof. Since $(\prod_i f_i) \circ \left(\prod_i F_i\right) = \prod_i (f_i \circ F_i) = \prod_i (K_i \circ g_i) = \left(\prod_i K_i\right) \circ \left(\prod_i g_i\right)$, the proof is complete. □

Here, we denote the category of soft modules as SM.

Proposition 3.10. *$\prod : \prod SM \to SM$ is a functor.*

Now, let the parameter set of $\{(F_i, A_i)\}_{i\in I}$ be a fixed point. We denote the fixed point of A_i as a_{0_i} and let $F_i(a_{0_i}) = 0$. For $A = \prod_{i\in I} A_i$ and $M = \oplus_{i\in I} M_i$, we define the mapping $F : A \to M$ by $F(a) = \oplus_{i\in I} F(a_i)$, for all $a = \{a_i\} \in A$. Then, (F, A) is a soft module over M.

Definition 3.13. (F, A) is said to be direct sum of $\{(F_i, A_i)\}_{i\in I}$ and denoted as $\oplus_{i\in I}(F_i, A_i)$.

The mapping $\varphi_j : A_j \to \prod_{i\in I} A_i$ is defined by $\varphi_j(a_j) = \{a_i\}$ such that if $i \neq j$, then $a_i = a_{0_i}$ and if $i = j$, then $a_i = a$. Also for embedding mapping $q_j : M_j \to \oplus_{i\in I} M_i$, $(q_j, \varphi_j) : (F_j, A_j) \to (F, A)$ is a soft homomorphism of soft modules.

Proposition 3.11. *Let $\{(F_i, A_i)\}_{i\in I}$ and $\{(H_i, B_i)\}_{i\in I}$ be family of soft modules over $\{M_i\}_{i\in I}$ and $\{N_i\}_{i\in I}$, respectively, and let $(f_i, g_i) : (F_i, A_i) \to (H_i, B_i)$ be a soft homomorphism of soft modules. Then $g_i : (A_i, a_{0_i}) \to (B_i, b_{0_i})$ is a mapping of fixed pointed set, i.e., $g_i(a_{0_i}) = b_{0_i}$. Also $\left(\oplus_{i\in I} f_i, \prod_{i\in I} g_i\right) : \oplus_{i\in I}(F_i, A_i) \to \oplus_{i\in I}(H_i, B_i)$ is a soft homomorphism of soft modules.*

Proposition 3.12. *$\oplus : \prod SM \to SM$ is a functor.*

Theorem 3.13. *If $\{(F_i, A_j)\}_{i\in I}$ is a family of intuitionistic fuzzy soft modules over $\{M_i\}_{i\in I}$, then $\prod_{i\in I}(F_i, A_i)$ is an intuitionistic fuzzy soft module over $\prod_{i\in I} M_i$.*

Proof. Define $F : \prod_{i\in I} A_i \to \prod_{i\in I} M_i$ by $F(\{a_i\}) = (\vee_{i\in I} p_i^{-1}(F_i)_{a_i}, \wedge_{i\in I} p_i^{-1}(F_i)^{a_i})$, where $p_i : \prod_{i\in I} M_i \to M_i$ is a projection mapping. Since $(p_i^{-1}(F_i)_{a_i}, p_i^{-1}(F_i)^{a_i}) : \prod_{i\in I} M_i \to [0, 1]$ is an intuitionistic fuzzy soft module over $\prod_{i\in I} M_i$,for all $i \in I$, $(\vee_{i\in I} p_i^{-1}(F_i)_{a_i}, \wedge_{i\in I} p_i^{-1}(F_i)^{a_i})$ is also an intuitionistic fuzzy soft module over $\prod_{i\in I} M_i$. □

Theorem 3.14. *If $\{(F_i, A_j)\}_{i\in I}$ is a family of intuitionistic fuzzy soft modules over the family of modules $\{M_i\}_{i\in I}$, then $\oplus_{i\in I}(F_i, A_i)$ is an intuitionistic fuzzy soft module over $\oplus_{i\in I} M_i$.*

Proof. Define $F : \prod_{i\in I} A_i \to \oplus_{i\in I} M_i$ for all $\{a_i\} \in \prod_{i\in I} A_i$ by $F(\{a_i\}) = (\wedge_{i\in I} j_i(F_i)_{a_i}, \vee_{i\in I} j_i(F_i)^{a_i})$ where $j_i : M_i \to \oplus_{i\in I} M_i$ is an embedding mapping. Since $(j_i(F_i)_{a_i}, j_i(F_i)^{a_i})$ is an intuitionistic fuzzy soft submodule over $\oplus_{i\in I} M_i$ for all $i \in I$, $F(\{a_i\})$ is an intuitionistic fuzzy submodule over $\oplus_{i\in I} M_i$. □

Lemma 3.15. (1) *Given modules $\{M_i\}_{i\in I}$ and N and a family of R-homorphisms $\mathrm{A} = \{f_i : M_i \to N\}_{i\in I}$. If $\{(F_i, A_j)\}_{i\in I}$ are intuitionistic fuzzy soft modules over $\{M_i\}_{i\in I}$, then there exist an intuitionistic fuzzy soft module $\left(H, \prod_{i\in I} A_i\right)$ over N such that for all $i \in I$,*

$$f_i : (F_i, A_i) \to \left(H, \prod_{i\in I} A_i\right)$$

is an intuitionistic fuzzy soft homomorphism of intuitionistic fuzzy soft modules.

(2) *Given modules M and $\{N_i\}_{i\in I}$ and a family of R-homorphisms $\mathrm{B} = \{g_i : M \to N_i\}_{i\in I}$. If $\{(H_i, B_j)\}_{i\in I}$ are intuitionistic fuzzy soft modules over $\{N_i\}_{i\in I}$, then there exists an intuitionistic fuzzy soft module $\left(F, \prod_{i\in I} A_i\right)$ over M such that for all $i \in I$,*

$$g_i : \left(F, \prod_{i\in I} A_i\right) \to (H_i, B_i)$$

is an intuitionistic fuzzy soft homomorphism of intuitionistic fuzzy soft modules.

Proof. (1) Define $H : \prod_{i\in I} A_i \to N$ by $H(\{a_i\}) = (\vee_i f_i(F_i)_{a_i}, \wedge_i f_i(F_i)^{a_i})$.
(2) Define $F : \prod_{i\in I} A_i \to M$ by $F(a) = (\wedge_{i\in I} g_i^{-1}(F_i)_{a_i}, \vee_{i\in I} g_i^{-1}(F_i)^{a_i})$ for all $a = \{a_i\} \in \prod_{i\in I} A_i$. □

By using this lemma, we define the concepts of submodule, quotient module, product and coproduct operations in the category of intuitionistic fuzzy soft modules.

Corollary 3.1. *If (F, A) is an intuitionistic fuzzy soft module over M and N is a submodule of M and $i : N \to M$ is an embedding mapping, then $(i^{-1}(F), A)$ is an intuitionistic fuzzy soft module over N.*

Corollary 3.2. *If (F, A) is an intuitionistic fuzzy soft module over M and $p : M \to M/\sim$ is a canonical projection, then $(p(F), A)$ is an intuitionistic fuzzy soft module over quotient module $M/\sim$.*

If $\{(F_i, A_j)\}_{i\in I}$ is a family of intuitionistic fuzzy soft modules over the family of modules $\{M_i\}_{i\in I}$, then we can define the product and coproduct of these families by $\prod_{i\in I}(F_i, A_i)$ and $\oplus_{i\in I}(F_i, A_i)$, respectively.

Theorem 3.16. *The category of intuitionistic fuzzy soft modules has zero objects, sums, product, kernel and cokernel.*

Let M and N be, respectively, right and left modules over R(ring). Let (F, A) and (G, B) be two soft modules over M and N, respectively. We consider tensor product of modules as $M \otimes N$. The mapping

$$F \otimes G : A \times B \to M \otimes N$$

is defined by $(F \otimes G)(a, b) = F(a) \otimes G(b)$, for $\forall(a, b) \in A \times B$.

Proposition 3.17. *$(F \otimes G, A \times B)$ is a soft module over $M \otimes N$.*

Proof. For $\forall(a, b) \in A\times B$, $F(a)$, $G(b)$ are submodules of M and N, respectively. Since the module $F(a)\otimes G(b)$ is a submodule of $M \otimes N$, the proof is complete. □

Definition 3.14. $(F \otimes G, A \times B)$ is said to be tensor product of (F, A) and (G, B) and denoted by $(F, A) \otimes (G, B)$.

Now, let (F, A) and (G, B) be two intuitionistic fuzzy soft modules over M and N, respectively. We give the following mapping

$$F \otimes G : A \times B \to M \otimes N$$

by $(F \otimes G)(a, b) = (F_a \otimes G_b, F^a \otimes G^b)$, for $\forall (a, b) \in A \times B$ [15].

Theorem 3.18. *$(F \otimes G, A \times B)$ is an intuitionistic fuzzy soft module over $M \otimes N$.*

Proof. For $\forall (a, b) \in A \times B$, (M, F_a, F^a) and (N, G_b, G^b) are intuitionistic fuzzy soft modules. From [15] $(F_a \otimes G_b, F^a \otimes G^b)$ is an intuitionistic fuzzy submodule over $M \otimes N$. Then $(F \otimes G, A \times B)$ is an intuitionistic fuzzy soft module over $M \otimes N$. □

Definition 3.15. $(F \otimes G, A \times B)$ is said to be tensor product of (F, A) and (G, B), and denoted by $(F, A) \otimes (G, B)$.

4. Conclusion

This paper summarized the basic concepts of intuitionistic fuzzy soft sets and intuitionistic fuzzy soft modules. By using these concepts, we studied the algebraic properties of intuitionistic fuzzy soft sets in module structure. This work focused on intuitionistic fuzzy soft modules, intuitionistic fuzzy soft submodules, and intuitionistic fuzzy soft homomorphisms. To extend this work, one could study the properties of intuitionistic fuzzy soft sets in other algebraic structures such as rings and fields.

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Further reading

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