Abstract— In this paper, a systematic and efficient formulation for analysis of active circuits containing operational transconductance amplifiers (OTA) is presented. The modified nodal approach is used in obtaining the system equations of active circuits. The model of operational transconductance amplifier relating to the used analysis method is given. The model is a matrix-based approach. Thus, it allows computer-aided analysis of active circuits to be realized efficiently. One illustrative example is included into the study.

Index Terms— active circuits, modified nodal analysis, operational transconductance amplifier

I. INTRODUCTION

There are two basic methods relating to the representation of circuit equations: The state variables analysis and the modified nodal analysis (MNA). The state variables method, based on the graph theoretical approach, was developed before the MNA. It involves intensive mathematical process and has major limitations in the formulation of circuit equations. Some of these limitations arise because the state variables are capacitor voltages and inductor currents. Every circuit element can’t be easily included into the state equations. Especially, there are some restrictions in the analysis of active circuits. Because of the drawbacks of state variables analysis, the modified nodal analysis was first introduced by Ho et al. [1] and has been developed more by including many circuit elements (transformer, semiconductor devices, short circuit, etc.) into the system equations so far [2-4]. In this method, the system equations can be also obtained by inspection. Especially, it is very suitable for computer-aided analysis of active circuits. In this paper, it is shown how to include the model of operational transconductance amplifier (OTA), one of the basic elements of active circuits, into the MNA system.

The paper is organized as follows. In Section 2, the modified nodal analysis is explained. Section 3 summarizes the fundamental characteristics of OTA, before including it into the MNA system. In Section 4, we develop the MNA model of OTA. One application example of the approach is given in Section 5. The paper concludes in Section 6.

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II. SYSTEM EQUATIONS

The method used in the analysis of active circuits is very important for formulation of active elements. MNA system allows equations of active circuits to be easily and systematically obtained without any limitation. The modified nodal equations are given in t-domain, (1) and in s-domain, (2), as follows. In this method, the equations are first obtained in s-domain. Later, for analysis, they are transformed into t-domain or frequency domain.

\[ Gx(t) + \frac{d}{dt}x(t) = Bu(t) \]  \hspace{1cm} (1)

\[ GX(s) + sCX(s) = BU(s) \]  \hspace{1cm} (2)

Where G, C, B are coefficient matrices. All conductances and frequency-independent values arising in the MNA formulation are stored in matrix G, capacitor values which are frequency-dependent in matrix C. U(s) represents the source vector. The unknown vector X(s) contains both voltage and current variables. Taking into account the types of variables, the unknown vector is partitioned as follows.

\[ X(s) = \begin{bmatrix} X_1(s) \\ \vdots \\ X_2(s) \end{bmatrix} \]  \hspace{1cm} (3)

Here, \( X_1(s) \) represents nodal voltage variables, \( X_2(s) \) represents current variables relating to independent and controlled voltage sources, short circuit elements, etc. \( X_2(s) \) also expresses required additional equations in the formulation of MNA. If there are \( n \) nodes and \( m \) current variables in a circuit, \( X_1(s) \) vector contains \( n-1 \) nodal voltage variables except reference node (ground) and \( X_2(s) \) vector contains \( m \) current variables. Thus, the unknown vector \( X(s) \) contains \( k=n-1+m \) variables.

\[
X_1(s) = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_{n-1} \end{bmatrix}, \quad X_2(s) = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_m \end{bmatrix} = \Rightarrow
\]
Fig. 1. OTA device.

\[
X(s) = \begin{bmatrix}
X_1(s) \\
\vdots \\
X_{n-1}(s) \\
I_1 \\
\vdots \\
I_m
\end{bmatrix} = \begin{bmatrix}
U_1 \\
\vdots \\
U_{n-1} \\
\vdots \\
I_1 \\
\vdots \\
I_m
\end{bmatrix}
\]  

(4)

III. OTA MODEL

The OTAs, one of the linear active devices in present-day analog integrated circuit applications, have broader frequency band, electronic tunability of the transconductance and good stability characteristics. Hence the use of OTA is very common [6-9]. Before obtaining the MNA model of OTA, the fundamental properties of OTA should be summarized. The OTA device is ideally shown as in Fig.1. OTA has inverting and non-inverting inputs and an output, like a normal Op amp. \(g_m\) is the transconductance which is the ratio of the output current to the input voltage. \(g_m\) can be adjusted to the desired value with a control voltage applied from outside. This property makes OTA very convenient for design and implementation of any circuit. Therefore, it is very important to model the OTA for analysis of circuits including OTA as above. As it known, there are some differences between a physical OTA and the ideal OTA characteristics [6]. But, in this paper, we assume that the OTA is ideal because our goal is analysis of OTA circuits by MNA. In this case, the transconductance will be constant.

The input and output impedances in the model are assumed to be ideal values of infinity. It also contains the voltage controlled current source whose gain is \(g_m\). In the ideal OTA operating in the linear mode, \(I_o\) is limited, if it is desired to describe the output dynamics of the circuit, output saturation current \(I_s\) and output saturation voltage \(U_d\) must be taken consideration.

In the linear region, the input-output relation is

\[
I_o = g_m(U_p - U_n) = g_m U_d
\]  

(5)

It explains the relationships between the input voltages \((U_p, U_n)\) and the output current \((I_o)\). Since the input resistance of ideal OTA is infinite, the input currents must be zero.

\[
I_p = 0, \quad I_n = 0
\]  

(6)

IV. MNA MODEL OF OTA

The ideal OTA concept is a good approximation to analyze OTA circuits. Therefore, this concept is used for developing the MNA model of OTA. For MNA structure, first, the terminal equations of OTA (OTA constraints), given in (5) and (6), are expressed together as in (7).

\[
\begin{bmatrix}
I_p \\
I_n \\
I_o
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & U_p \\
0 & 0 & 0 & U_n \\
g_m - g_m & 0 & 0 & 0
\end{bmatrix}
\]  

(7)

As explained in Section 2, there are \(m\) current variables \((X_i(s))\) in MNA system. \(I_p\), \(I_n\) and \(I_o\) currents of OTA are located in \(X(s)\) vector.

An active circuit contains \(n\) nodes, including three terminals (nodes) of OTA. In MNA system, there are \(n-1\) nodal voltage variables \((X_i(s))\).

In (8), it is shown how to be included the terminal equations of OTA, input currents and output current property in (7), into MNA system in (2). The constraints of OTA are stored in matrices G and B. (8) gives the MNA model of OTA.
V. APPLICATION EXAMPLE

To show the MNA formulation of OTA, we use the active filter (low pass filter) circuit in Fig.3. The circuit has \( n_{ref} = 3 \) nonreference nodes, including terminals of OTAs. Thus, in the MNA system, \( X_1(s) \) vector contains 3 nodal voltage variables. The voltage and current constraints of OTA are included into the system equations, as shown in the MNA model of OTA in (8). The current variables in \( X_2(s) \) vector are \( I_{p1}, I_{n1}, I_{o1}, I_{U1} \) (source current). They’re relating to additional equations.

Nodal (main) equations in s-domain:

\[
\begin{align*}
\text{a} & \rightarrow \quad I_U + I_{pl1} = 0 \\
\text{b} & \rightarrow \quad I_{p2} - I_{o2} + sC_1U_b = 0 \\
\text{c} & \rightarrow \quad I_{n1} + I_{n2} + sC_2U_o - I_{o2} = 0
\end{align*}
\]

Additional equations:

1st OTA \( \rightarrow \) \( I_{pl} = 0, I_{n1} = 0, I_{o1} - g_{m1}(U_a - U_c) = 0 \)

2nd OTA \( \rightarrow \) \( I_{p2} = 0, I_{n2} = 0, I_{o2} - g_{m2}(U_b - U_c) = 0 \)

\( U_a = U \)

The overall equations constitute the MNA system (9). They are represented in matrix form, as in (2). The MNA model of OTA, given by (8), can be also seen from system equations in (9).

The system model in (9) can be systematically obtained by inspection because of the advantages of MNA and the MNA model of OTA. By using this system model, the desired analysis of active circuit, such as t-domain or frequency domain, can be obtained. In the MNA system given by (9), the current constraints of the ideal OTA concept, \( I_p = I_n = 0 \), appear to be fairly useless because it draws no currents at its inputs. Therefore, these currents are ignored when formulating the MNA in order that the system matrix has min. dimensions.

After solving the system equations (9), the voltage transfer function is obtained as follows.

\[
H(s) = \frac{U_a(s)}{U(s)} = \frac{U_c(s)}{U(s)} = \frac{g_{m1}g_{m2}}{g_{m1}g_{m2} + sC_1g_{m2} + s^2C_1C_2}
\]

VI. CONCLUSION

An efficient and systematic approach for analysis of active circuits containing OTA has been presented in this paper. The modified nodal approach, suitable for computer-aided analysis of active circuits, is used in the system analysis. The fundamental characteristics of OTA have been summarized and the MNA model of OTA has been developed. As a result, a matrix-based framework for computer-aided analysis of active circuits has been formulated. The method is general and can be applied to all possible active circuits structures. One example is included to show the efficiency of the analysis method and the MNA model of OTA.
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